A Fokker-Planck model for nonlocal impact ionization in semiconductors

Biju Jacob, P. N. Robson, J. P. R. David and G. J. Rees
Department of Electronic and Electrical Engineering
University of Sheffield, Mappin Street, Sheffield S1 3JD, United Kingdom.

Avalanche Photodiodes (APDs) have the advantage over conventional p-i-n detectors because of their internal gain. However, this comes at the expense of increased signal to noise ratio because of the excess noise due to the inherently random phenomenon of impact ionization leading to carrier multiplication. Recently there has been increased interest in APDs with thin, sub-micron, multiplication regions, using operating electric fields as high as 1 MV/cm, that have significantly lower excess noise than their thicker counterparts for the same value of multiplication. This reduction in noise at high fields has been attributed to the more deterministic ionization process at work in these devices [1]. The Probability Distribution Function (PDF) for impact ionization path length is a crucial quantity for understanding and modelling the low noise behaviour of these devices, in which the high electric fields needed to produce avalanche gain narrow the PDF, thereby reducing the randomness in ionization position and hence the noise in the multiplication. In this paper we present a method for calculating PDFs using a Fokker-Planck model.

The central problem of carrier transport in semiconductors is to solve the Boltzmann Transport Equation (BTE) to obtain the carrier momentum distribution function. Under the condition in which the electrons *drift*, (which is valid in APDs), the BTE can be approximated by a simpler equation of a multivariate Fokker-Planck Equation (FPE) type which can be solved to obtain the carrier distribution in energy and position [2, 3]. A parabolic energy band model with only momentum randomizing optical phonon deformation potential scattering is chosen for the present purpose. In the hard threshold impact ionizing scheme when the energy of the electron reaches the threshold energy, E_T, it immediately undergoes impact ionization. This is incorporated into the present model by setting an "absorbing boundary" at $E = E_T$, which acts as a sink for the electrons whose energy exceeds the threshold. Fig. 1 shows the steady state evolution of electron distribution in energy-position space owing to the drift and diffusive forces, inherent in the Fokker-Planck scheme. When the distribution is incident on the absorbing boundary it immediately loses electrons due to impact ionization. The number of electrons crossing the threshold per unit length at position x, the electron energy flux, is the desired ionization path length PDF $h(x)$.

For a chosen electric field the PDF was obtained by computing the electron energy flux at the hard threshold boundary. This was compared with the PDF obtained from an equivalent Monte-Carlo simulation [4] (fig. 2). It can be seen that the Fokker-Planck model correctly reproduces the non-local nature of the ionization path length PDF. A quantitative comparison between the Fokker-Planck and Monte-Carlo PDFs is given in fig. 3, where the mean ionization path length, $<x>$, and the coefficient of variation ($\sqrt{(<x^2>/<x>^2)-1}$), are plotted against electric field.

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[2] E. Bringuier, Phys. Rev. B **57**, 2280 (1998)
[3] E. Bringuier, Phys. Rev. B **56**, 5328 (1997)
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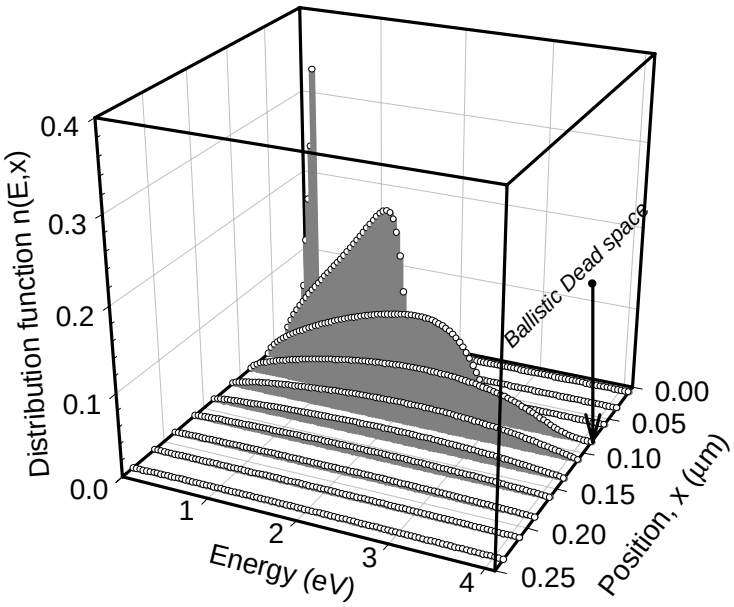

FIG. 1 Electron distribution *n(E,x)* computed using the Fokker Planck model at 500kV/cm with optical phonon energy = 29 meV, momentum mean free path = 61Å and E_T = 4.1 eV.

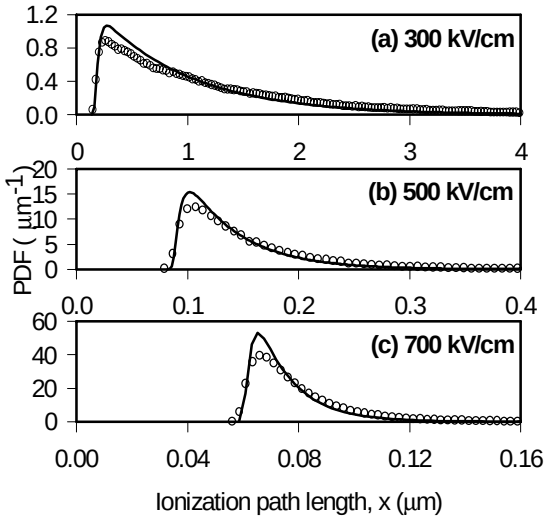

FIG. 2 Ionization path length PDFs from Fokker-Planck (lines) and Monte-Carlo (symbols) plotted at electric fields of (a) 300 kV/cm, (b) 500 kV/cm and (c) 700 kV/cm.

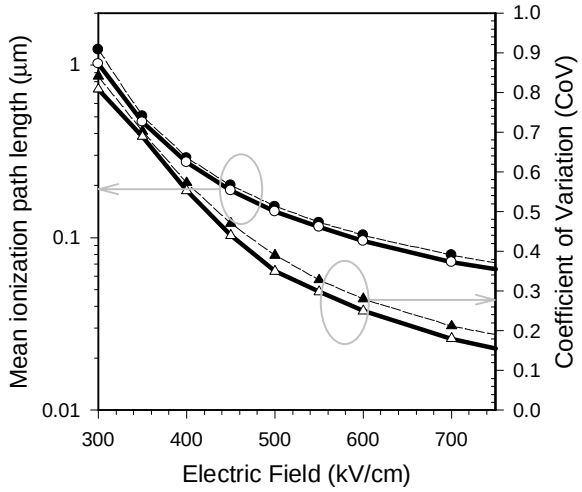

FIG. 3 Mean ionization path length as a function of electric field: Fokker-Planck (open circles) and Monte-Carlo (filled circles). Also shown is the coefficient of variation: Fokker-Planck (open triangles) and Monte-Carlo (filled triangles).