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# Two Models for Electro-Magnetic Wave Amplifier by Utilizing Traveling Electron Beam.


**by,**

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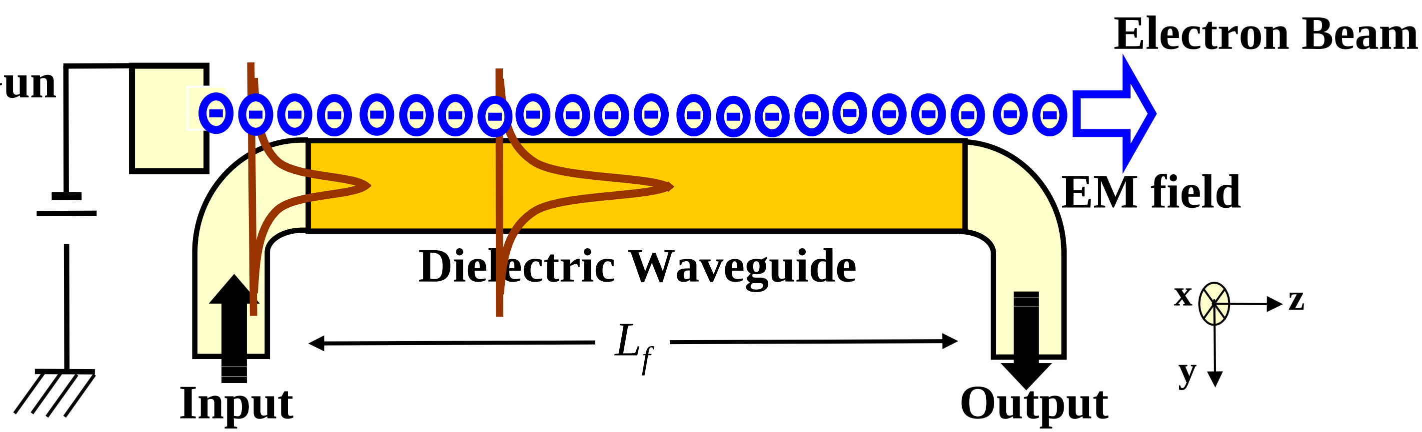
# Headlines

- **1-General Scheme of Electro-Magnetic Wave Amplifiers.**
- **2-Theoretical Models of Amplification Mechanism.**
  - **2.A- *Coherent Electron Wave (CEW) Model.***
  - **2.B- *Localized Electron (LE) Model.***
- **3-Thermal Effect on the Amplification Gain.**
- **4- Experimental Evidences.**

What is the Electro-Magnetic wave Amplifiers?

# Electro-Magnetic (EM) wave Amplifiers

➢Scheme of EM wave amplifier is same from microwave region to X-ray region.




**Energy exchange between electron beam and EM-wave.**

➢If the electric field component

$$E = F(z)T_z(x,y)e^{j(\omega t-\beta z)} \rightarrow (1)$$

*F(z) is field amplitude in z-direction and $T_z(x,y)$ is transverse field distribution.*

The amplification gain (g) is,

$$\frac{\partial F(z)}{\partial z} = \frac{g}{2}F(z) \quad \rightarrow \quad (2)$$

# Amplification Models

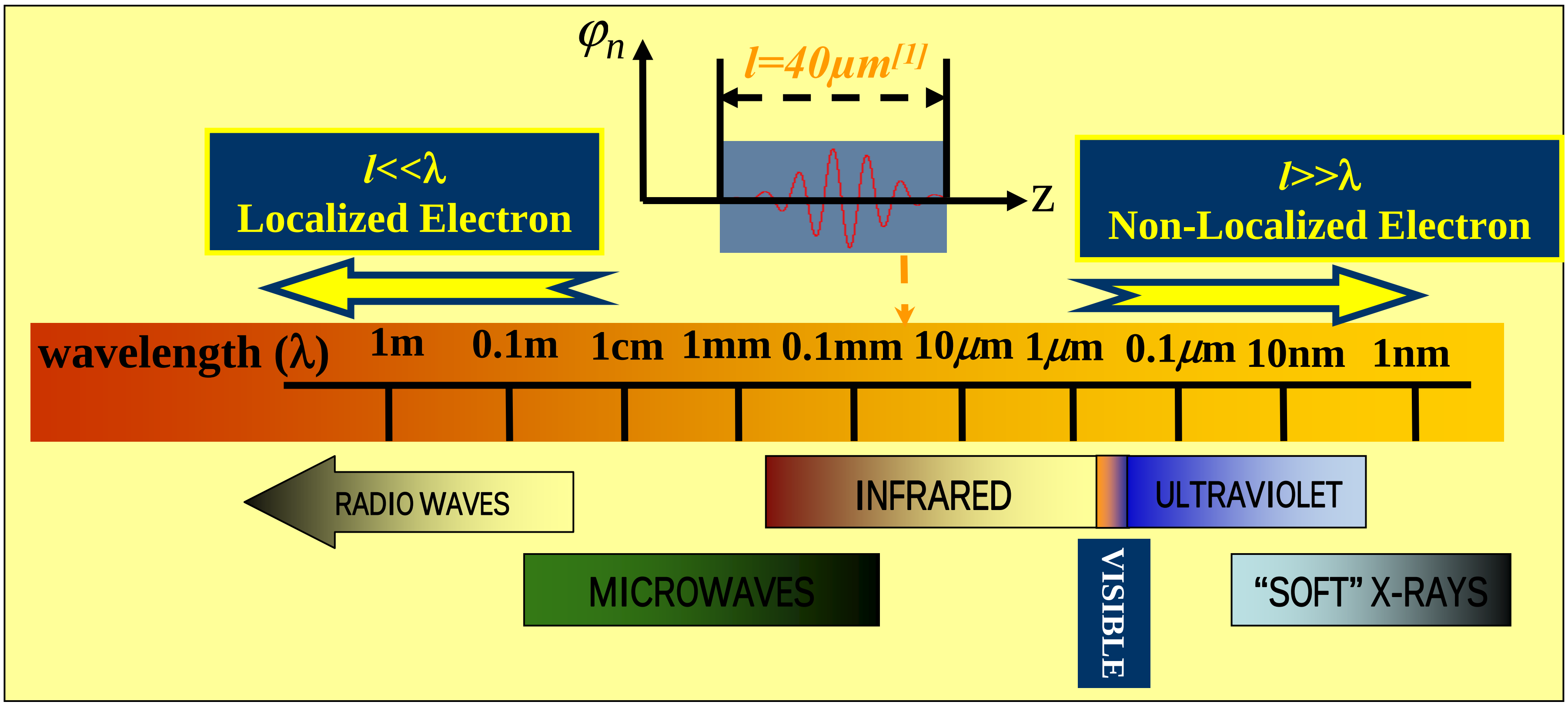
**“How does the electron see the EM-wave”?**

# How does the Electron see the Electromagnetic wave?

## ➢Form of the electron wave function:

$$\varphi_n(r) = \frac{1}{\sqrt{\ell^3}} e^{jk_n z} \rightarrow (3)$$

**$k_n$: the electron wave number at *n-th* level .**
**$l$ : the coherent length of electron wave "electron size".**

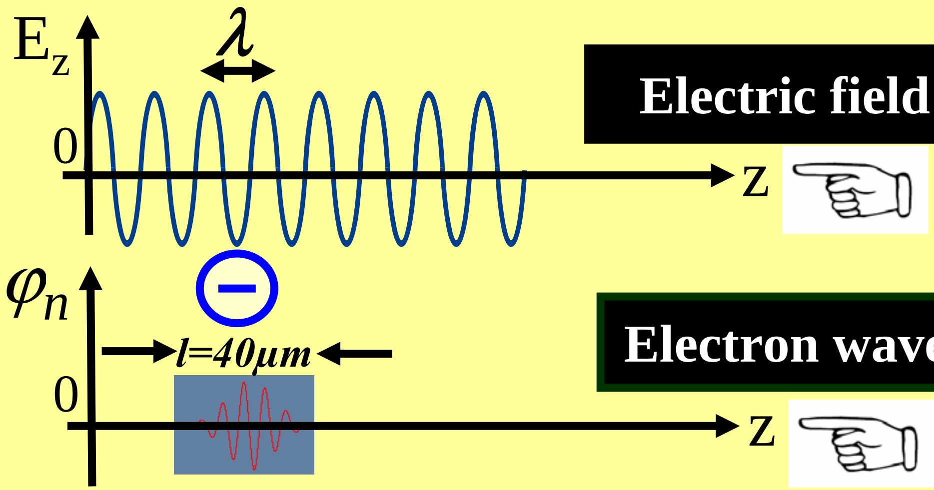




**[1] Y. Kuwamura, M. Yamada, R. Okamoto, T. Kanai and H. Fares, "Observation of TM guided spontaneous emission in high refractive index optical waveguide excited by the traveling electron beam" Proc. 8th Int CLEO/QELS Conf. San Jose, CA, USA, May. 2008.**

# Models of amplification gain analysis:

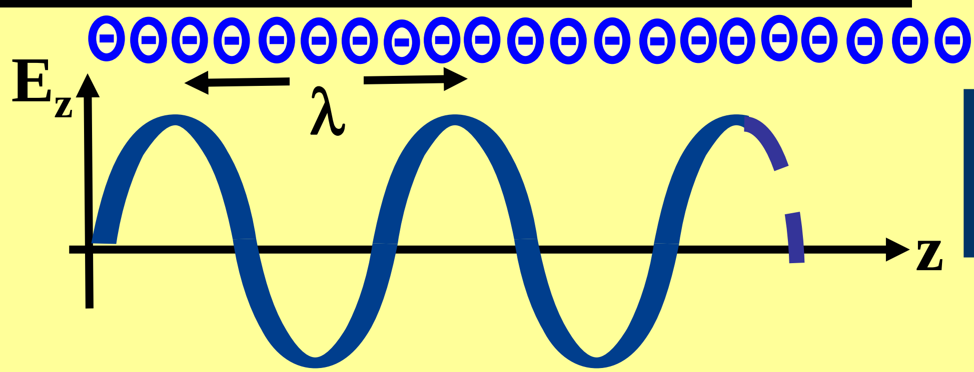
## 1- Coherent Electron Wave Model ($l >> \lambda$)




**A- Electron wave picture.**
**B- Quantum mechanical trend.**

**Electron wave size ($l$) > $\lambda$.**

## 2- Localized Electron Model ($l << \lambda$)




**A- Electron particle picture.**
**B- Classical mechanical trend**

**Electron wave size ($l$) < $\lambda$, electrons density modulation is shown.**

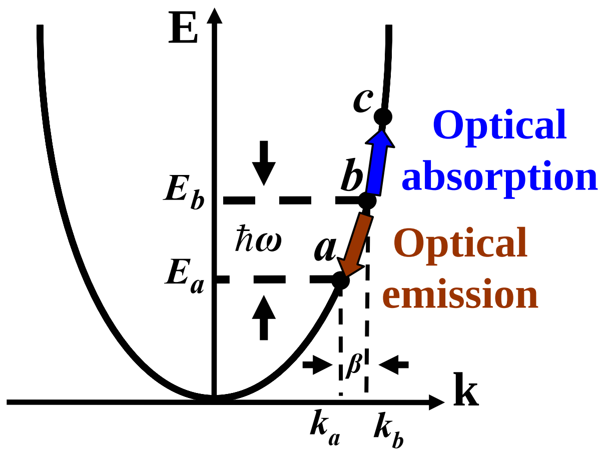
# First Model
# “Coherent Electron Wave Model”

# 1- Physical interpretation of amplification

## The electron transits basing on the rules of:

$$E_b - E_a = \hbar\omega \quad \text{"Energy consevation"}$$
$$k_b - k_a = \beta \quad \text{"Momentum consevation"}$$

$E_i$ is the electron energy at level *i (i=a or b or c)*.
$k_i$ is the electron wave number at level *i*.
ω is emitted EM-wave frequency.
$\beta$ is EM-wave propagtion constant



Energy diagram of the electron transition

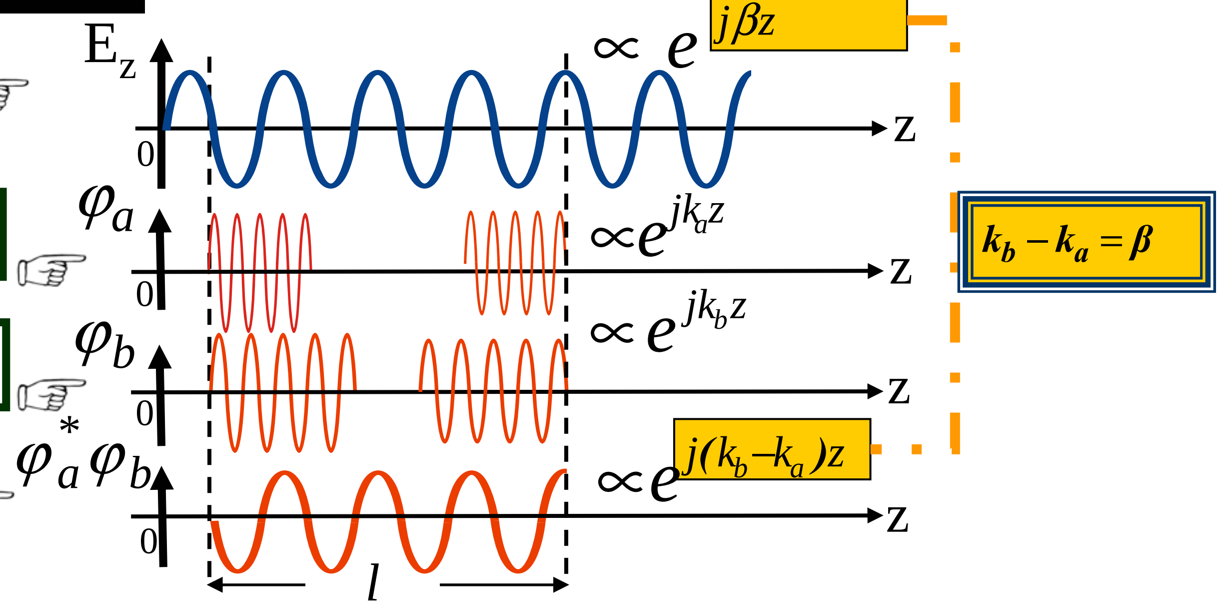
## The condition of amplification

Electric field

Electron wave at final level

Electron wave at initial level

Mixed electron wave




**Spatial variation coincidence corresponds to momentum conservation**

## 2- Gain coefficient in CEW-Model

- By some tools of statistical quantum mechanics,

$$g \propto \left| \langle \phi_a \left| T(x,y) e^{-j\beta z} \right| \phi_b \rangle \right|^2 \rightarrow (4)$$

**Finally, the expression of amplification gain in CEW-Model**

$$g(v_b, v_{em}) = \sqrt{\frac{\mu_o}{\varepsilon_o}} \frac{e J_o \tau v_b}{n_{eff} \hbar \omega} \xi \times D(v_b, v_{em}) \rightarrow (5)$$

$$, \xi = \iint_S | T_z(x,y) |^2 dx\, dy$$
*(Coupling coefficient)*

*$v_b$ is electron velocity influenced by applied voltage $V_b$.*
*$v_{em} = c/n_{eff}$ is EM-wave phase velocity.*
*$J_0$ is average electron beam current density and $\tau$ is electron relaxiation time.*

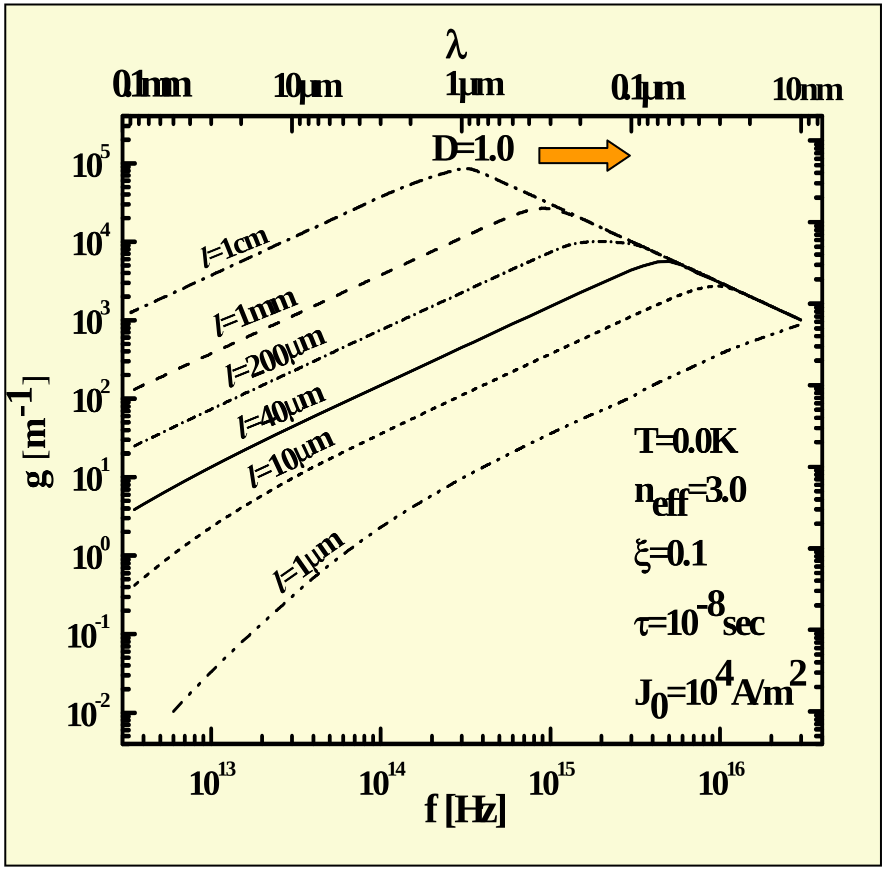
- *D is the dispersion function controlling the gain profile,*

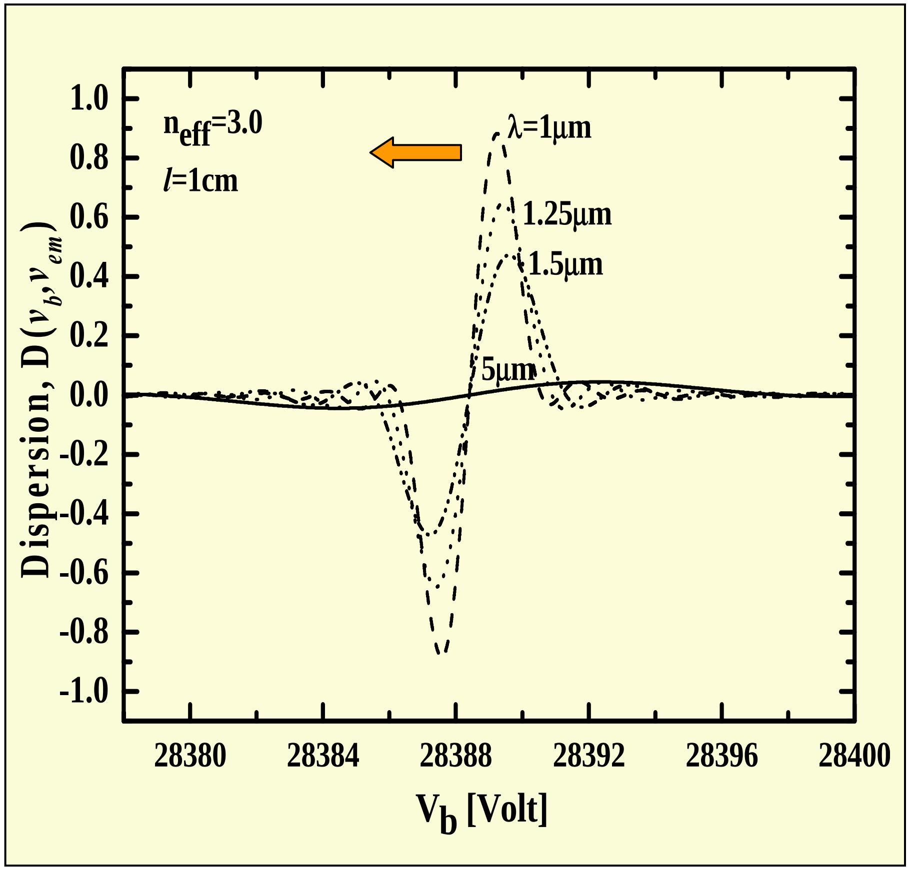
$$D(v_b, v_{em}) = \left\{ \begin{array}{l} Sinc^2 \left[ \left\{ \frac{\sqrt{2m_o}}{\hbar} \left( \sqrt{eV_b} - \sqrt{eV_b - \hbar\omega} \right) - \frac{n_{eff}\omega}{c} \right\} \frac{\ell}{2} \right] \\ - Sinc^2 \left[ \left\{ \frac{\sqrt{2m_o}}{\hbar} \left( \sqrt{eV_b + \hbar\omega} - \sqrt{eV_b} \right) - \frac{n_{eff}\omega}{c} \right\} \frac{\ell}{2} \right] \end{array} \right\} \rightarrow (6)$$

# Gain behavior with frequency variation in CEW-Model.

## The gain peak is affected by saturation of the dispersion function.

$$g(v_b, v_{em}) = \sqrt{\frac{\mu_o}{\varepsilon_o}} \frac{e J_o \tau v_b}{n_{eff} \hbar\omega} \xi \times D(v_b, v_{em})$$




**Variation of gain peak with EM frequency**

**Saturation of dispersion function to 1.**

# Second Model
# “Localized Electron Model”

# 1- Physical interpretation of amplification

**Synchronization condition,**

$$v_{el} \approx v_{em} \quad \text{where,} \quad v_{em} = \frac{\omega}{\beta}$$

**Electric field**

☞

**Electron wave at $\nu$ -level**

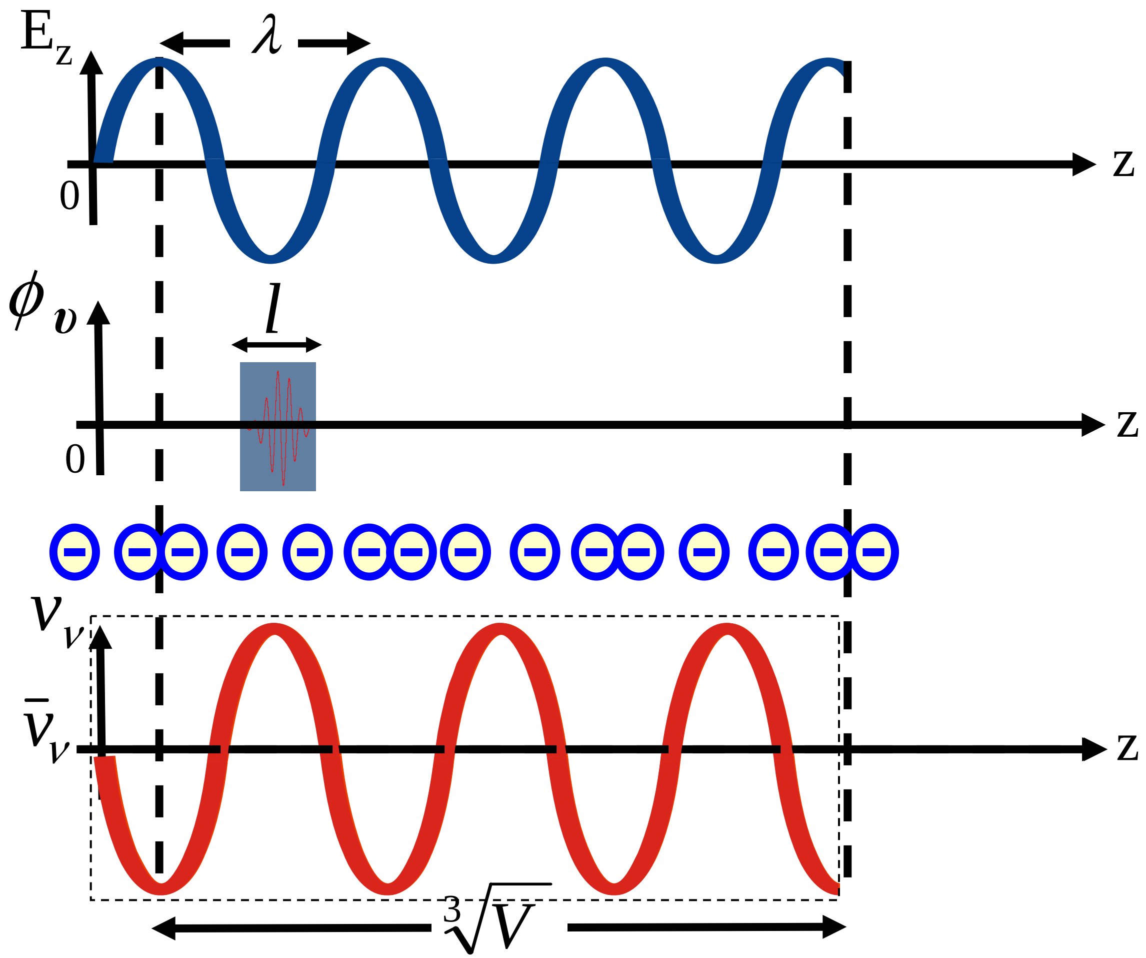
☞

**Electron density modulation**

☞

**Electron velocity modulation**

☞




**One synchronize wave modulates the electron velocity to start the amplification.**

## 2- Gain coefficient in LE-Model

- From the quantum mechanics point of view,

$$\tilde{\Psi} = \sum_{v} C_v \Psi_v \rightarrow (7)$$ (Total wave function)

*$\Psi_v$ is the wave function of v - electron*

- The form of velocity-modulation, (same as classical form)

$$\frac{\partial v_v}{\partial t} + \bar{v}_v \frac{\partial v_v}{\partial z} = -\frac{e}{m_o}\left\{F(z)T_z(x,y)\,e^{j(\omega t - \beta z)} + c.c\right\} - \frac{v_v - \bar{v}_v}{\tau} \rightarrow (8)$$

## Finally, The expression of amplification gain in LE-Model

$$g(v_b, v_{em}) = \xi \frac{e \mu_o J_o}{m_o} \times Y(v_b, v_{em}) \rightarrow (9)$$

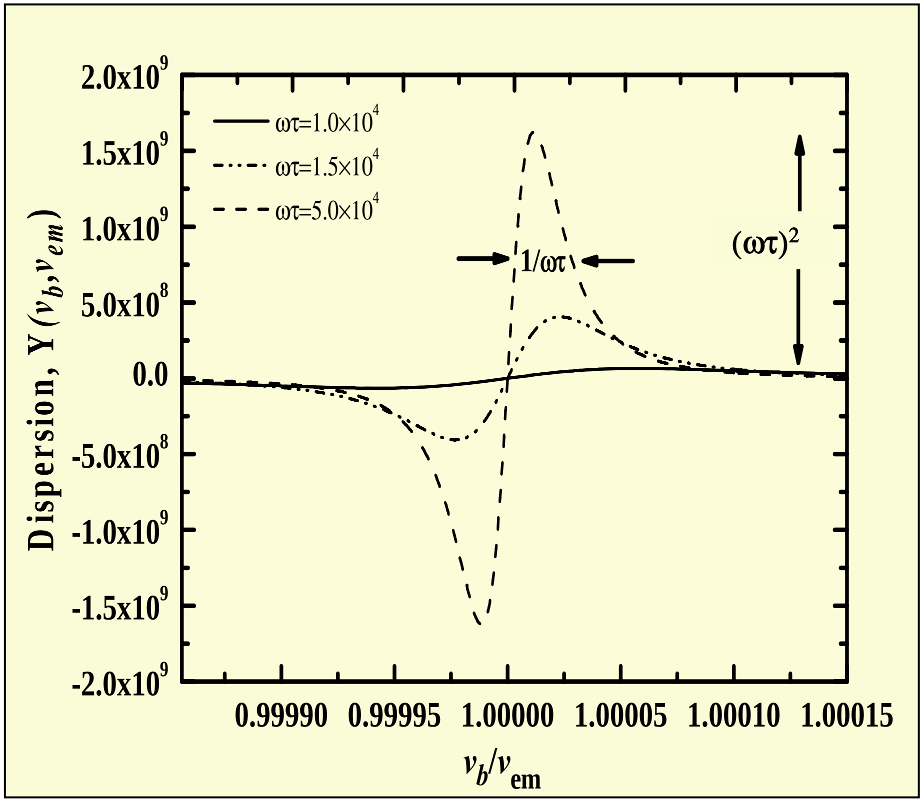
$Y(v_b, v_{em})$ is dispersion function controls gain profile,

$$Y(v_b, v_{em}) = Re\left\{\left(j + \frac{1}{\omega\tau}\right) \Big/ \left(\frac{n_{eff}}{c} v_b - 1 + \frac{j}{\omega\tau}\right)^2\right\}, v_{em} = \frac{c}{n_{eff}} \rightarrow (10)$$

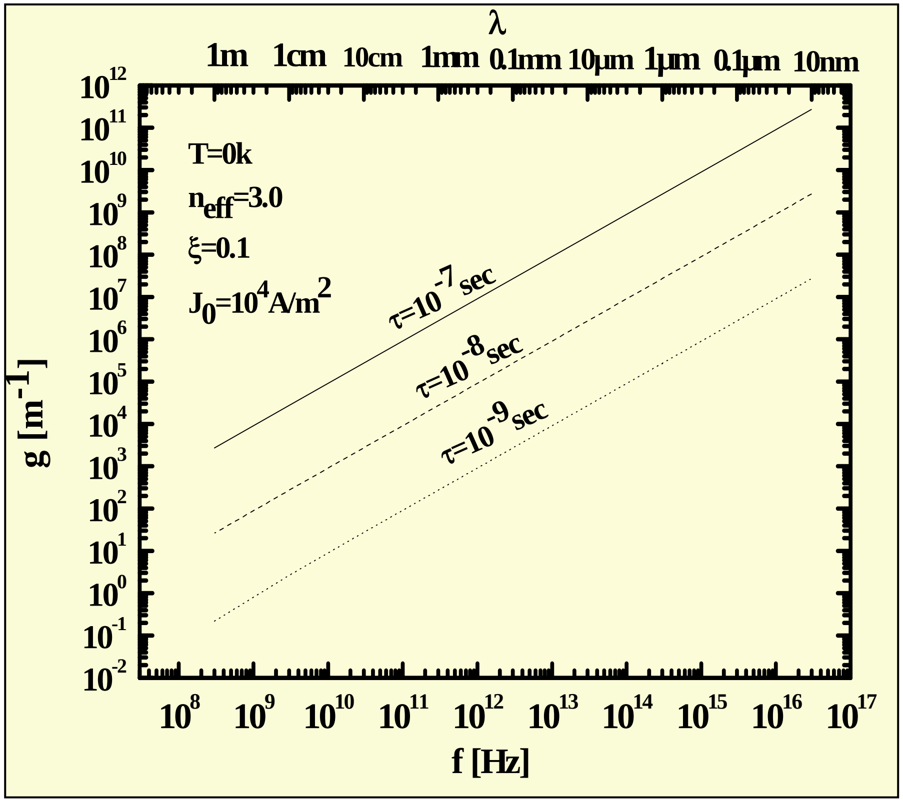
# Gain behavior with frequency in Localized Electron Model

**The gain increases infinitely with frequency, thermal effect limits this behavior.**

$$g(\nu_b,\nu_{em}) = \xi \frac{e\mu_o J_o}{m_o} \times Y(\nu_b,\nu_{em})$$




Dispersion function in gain coefficient by the LE-Model.




Variation of gain coefficient with the EM frequency by the LE-Model.

# Thermal Effect on the Amplification Gain

## Real gain with thermal effect "velocity broadening around the average value",

$$g(\bar{v}, v_{em}) \approx \int_o^\infty f(v_b, \bar{v})\, g(v_b, v_{em})\, dv_b \rightarrow (11)$$

$f(v_b, \bar{v})$ is the normalized Maxwell-Boltzmann distribution function.

**Where,**

$$f(v_b, \bar{v}) = \sqrt{\frac{m_o}{2\pi K_B T}}\, exp\left[-\frac{eV_b}{K_B T}\left(\frac{\bar{v}}{v_b} - 1\right)^2\right]$$

*$\bar{v}$ is the average electron velocity and $v_b$ is the real electron velocity.*

*$K_B$ is the Boltzmann constant and T is the absolute temperature.*

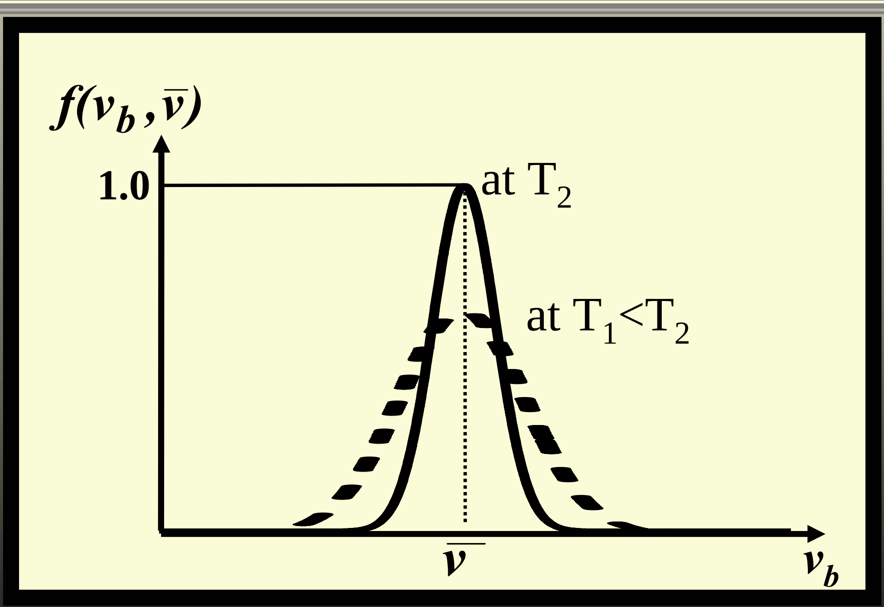
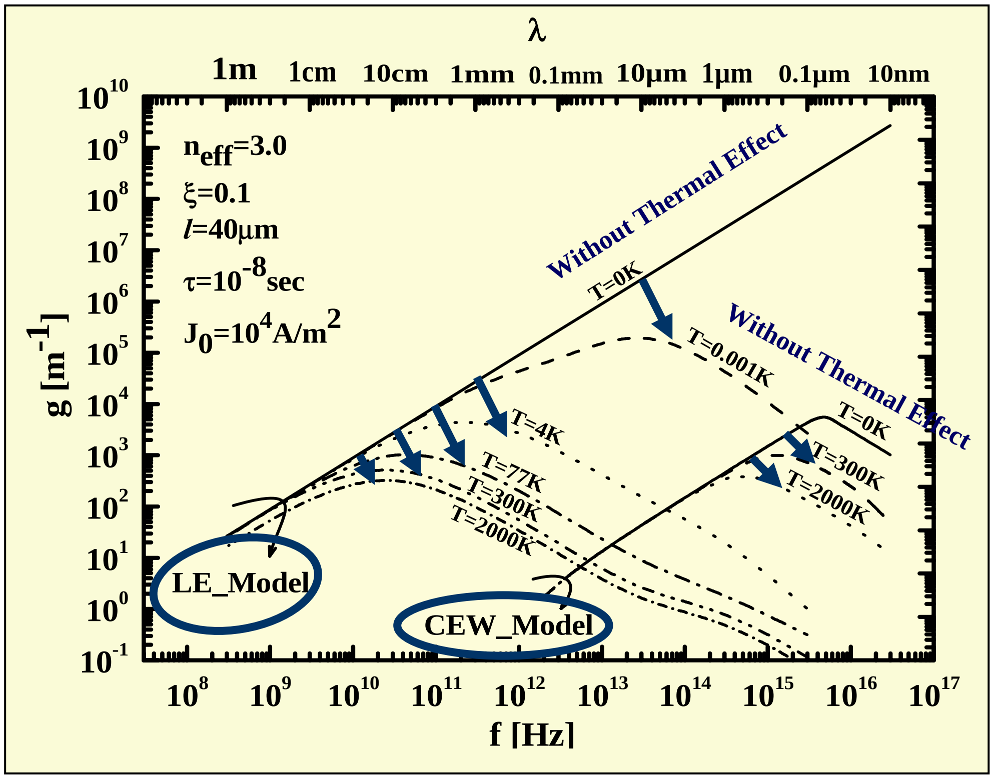


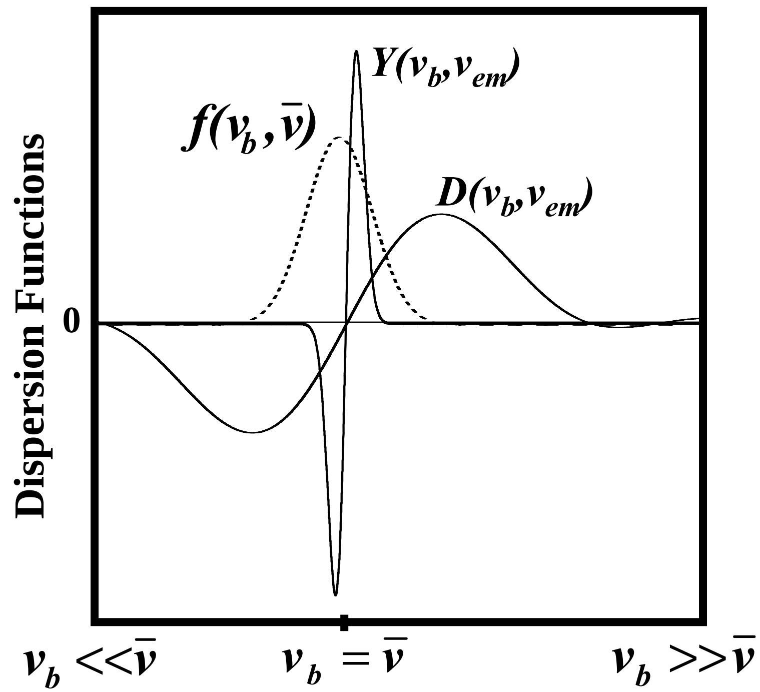

**Illustration of thermal distribution function.**

# The effect of thermal velocity broadening on gain amplification.

$$g(\bar{v}, v_{em}) \approx \int_o^\infty f(v_b, \bar{v})\, g(v_b, v_{em})\, dv_b$$

## The boundary between two models within THz region.



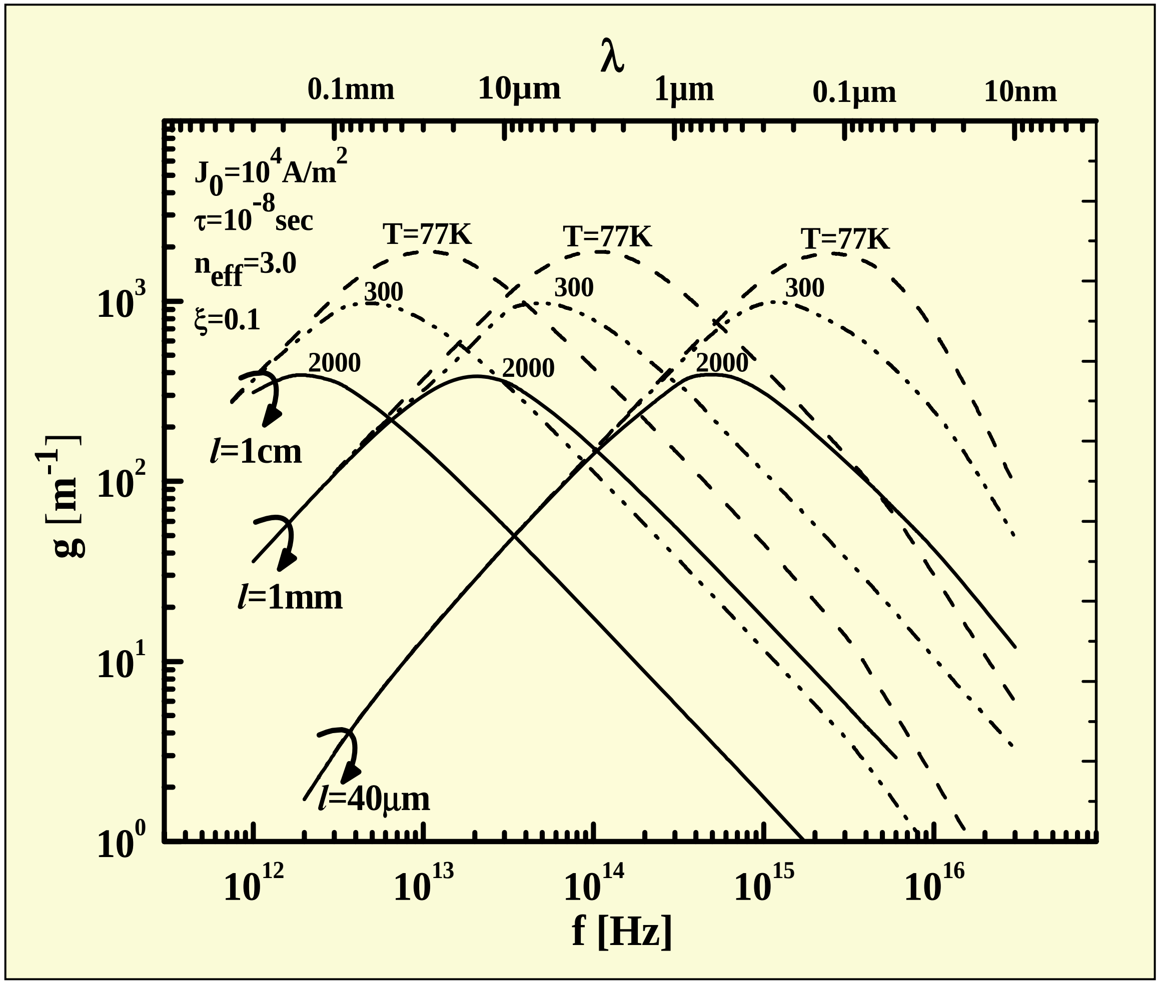

$v_b << \bar{v}$  $v_b = \bar{v}$  $v_b >> \bar{v}$

Illustration for dispersion relations

Variation of the peak values of gain coefficient with EM frequency by the CEW-Model and the LE-Model for several temperatures.

# The effect of electron size on the gain amplification

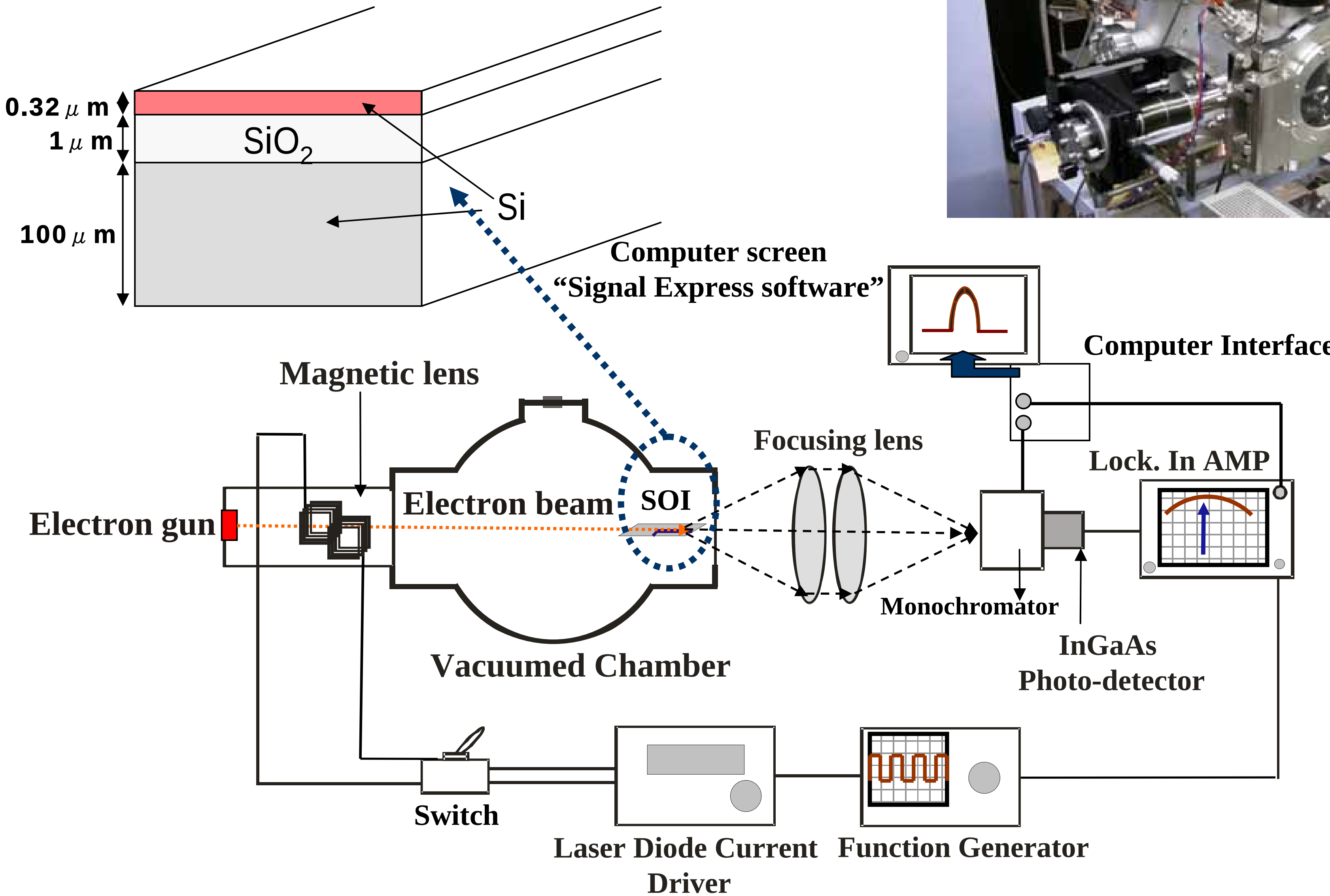
## The thermal effect gives same gain peaks for different coherence length.

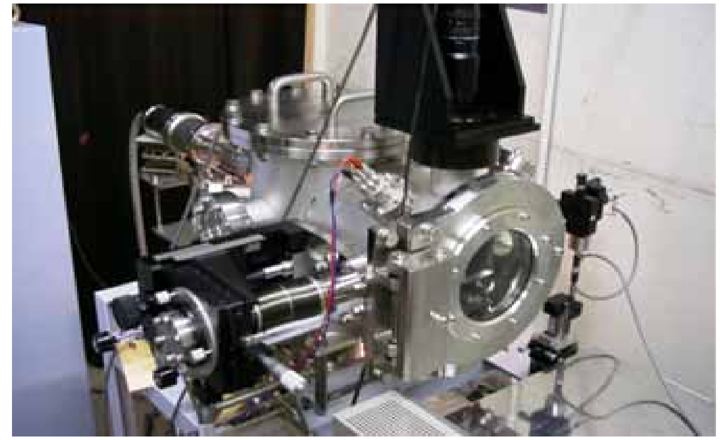



**Gain with different assumed coherence length at different temperature.**

# Experimental Evidences

# Experimental Setup



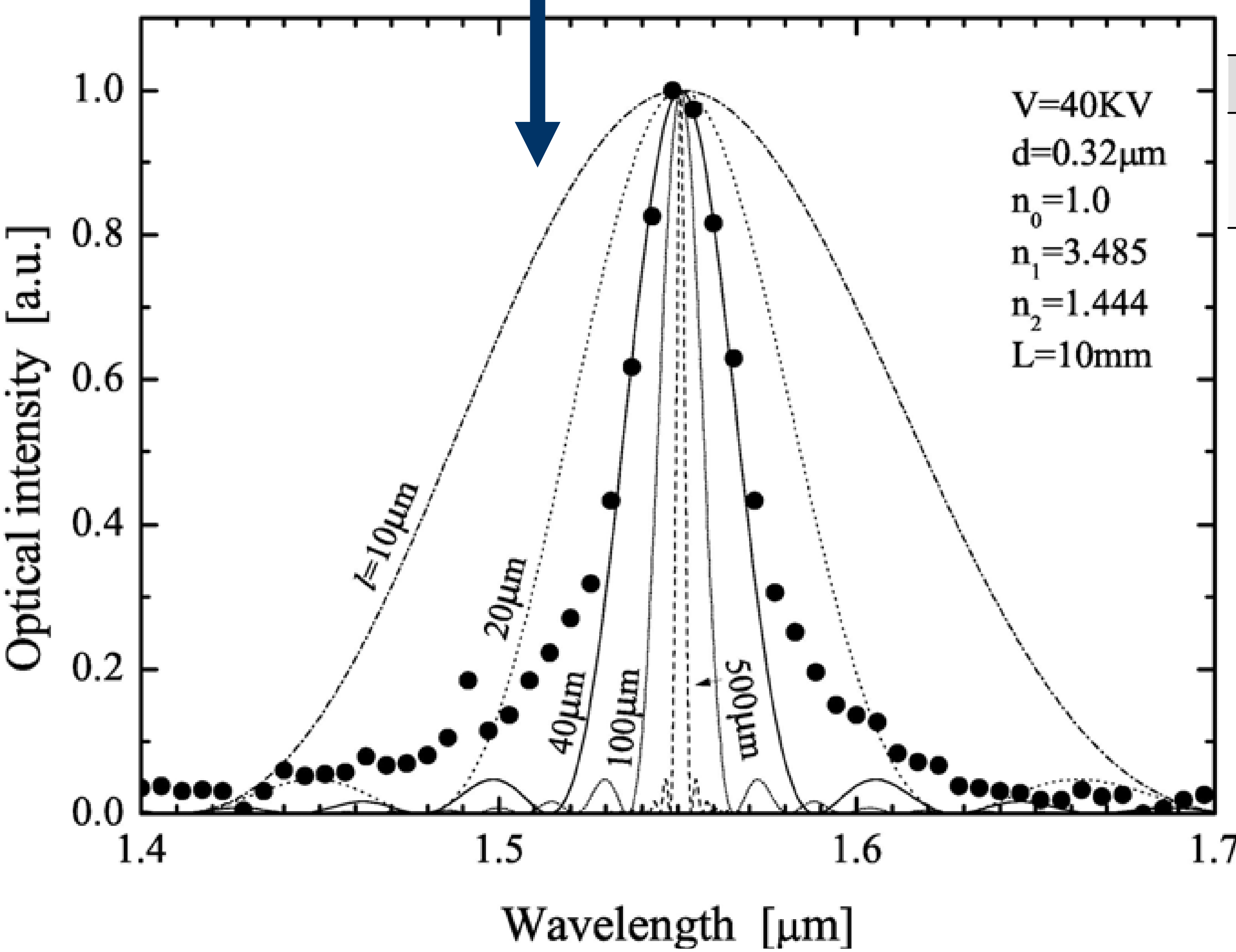



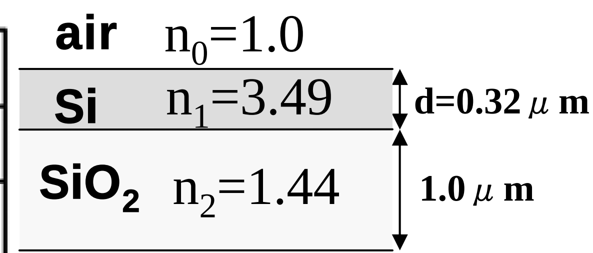
Experiment Setup

# Comparison of the emission profile with theoretical Calculation

$$g(v_b, v_{em}) \propto Sinc^2\left[\left\{\frac{\sqrt{2m_o}}{\hbar}\left(\sqrt{eV_b}-\sqrt{eV_b-\hbar\omega}\right)-\frac{n_{eff}\omega}{c}\right\}\frac{\ell}{2}\right]+Sinc^2\left[\left\{\frac{\sqrt{2m_o}}{\hbar}\left(\sqrt{eV_b+\hbar\omega}-\sqrt{eV_b}\right)-\frac{n_{eff}\omega}{c}\right\}\frac{\ell}{2}\right]$$

**EMISSION** **ABSORPTION**

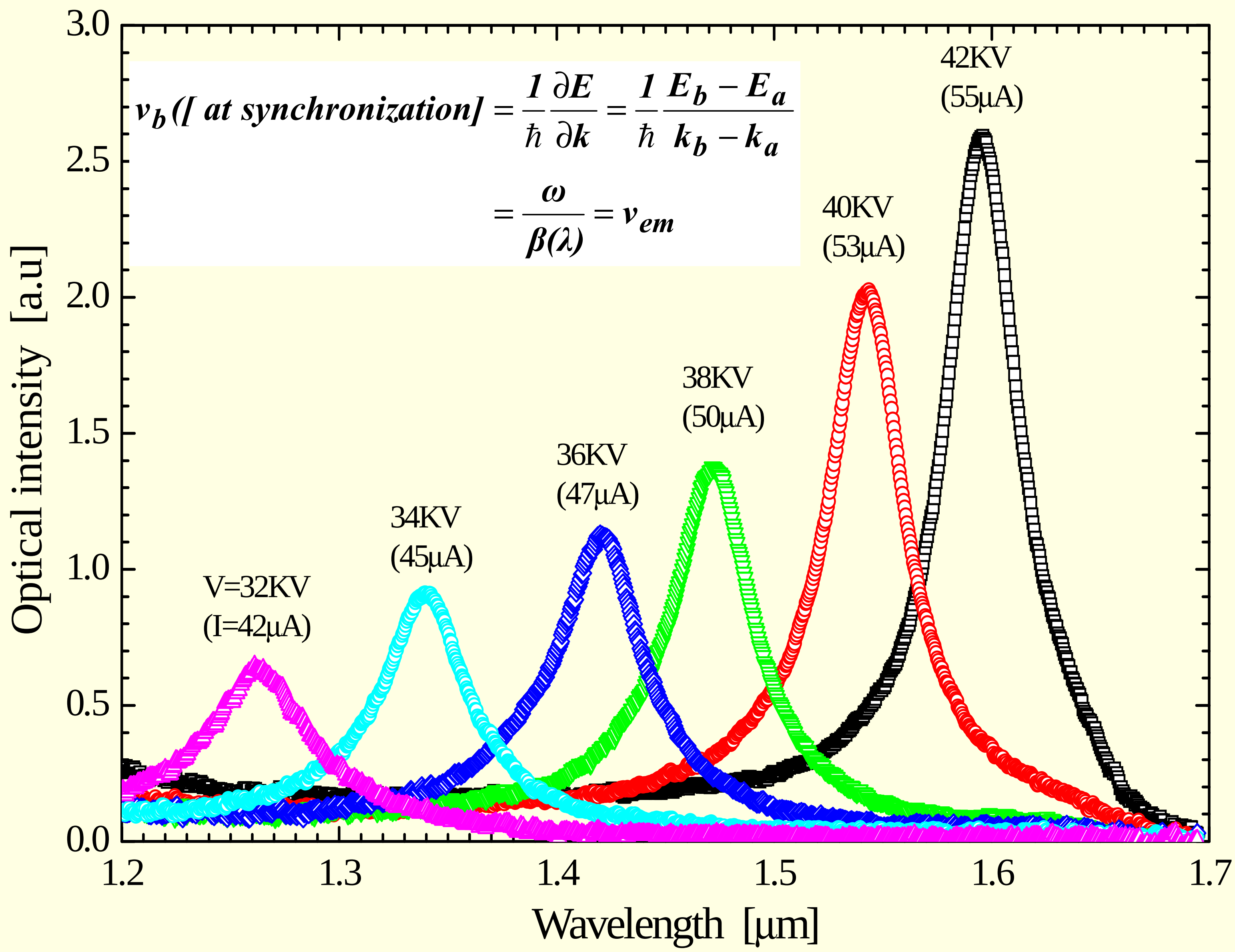



$$\ell = \frac{1}{N^{1/3}}$$

$N$ is electrons density

# Emission spectrum for different acceleration voltage



# THANKS FOR YOUR ATTENTION