Dispersive Contour-Path Algorithm for the TRC-FDTD Method

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***Abstract*—We introduce the dispersive contour-path (DCP) algorithm to the FDTD method based on the trapezoidal recursive convolution (TRC) technique. The scattering efficiency is evaluated in the analysis of a metallic nanocylinder. It is found that the DCP-FDTD method yields faster convergence in the calculation of the scattering efficiency, when compared to the standard FDTD method with a staircase approximation. The memory usage of the TRC-DCP-FDTD method is reduced in comparison with that of the Z-transform FDTD method.**

I. Introduction

The finite-difference time-domain (FDTD) method has widely been used for the numerical analysis of electromagnetic problems [1]. The conventional FDTD method is based on the Cartesian coordinate system. Therefore, it is difficult to accurately handle curved interfaces. An improved method has been studied for handling curved interfaces. For example, Mohammadi *et al*. have extended a contour-path effective-permittivity [2] to metal-dielectric interfaces of arbitrary shape, which is named as a dispersive contour-path (DCP) FDTD method. They have formulated the DCP-FDTD method based on the Z-transform [3], [4]. It has been shown that spurious surface plasmon resonances due to the staircase approximation can be eliminated using the DCP-FDTD method.

In this article, we introduce the DCP algorithm to the FDTD method based on the trapezoidal recursive convolution (TRC) technique [5]-[7], which is much simpler than the Z-transform. The scattering efficiency is evaluated in the analysis of a metallic nanocylinder. The results of the TRC-DCP-FDTD method are in excellent agreement with those of the Z-transform DCP-FDTD method, with reduced computational requirements.

II. Discussion

We formulate the two-dimensional DCP-FDTD method based on the TRC technique. The frequency-dependent nature of the metal permittivity is often expressed by the Drude model as

$$\varepsilon_r(\omega) = \varepsilon_\infty + \frac{\omega_{\mathrm{p}}^2}{\omega(j\nu_{\mathrm{c}} - \omega)} \tag{1}$$

where ε_∞ is the dielectric constant of the material at infinite frequency, ω is the angular frequency, ω_{p} is the electron plasma frequency and ν_{c} is the effective electron collision frequency.

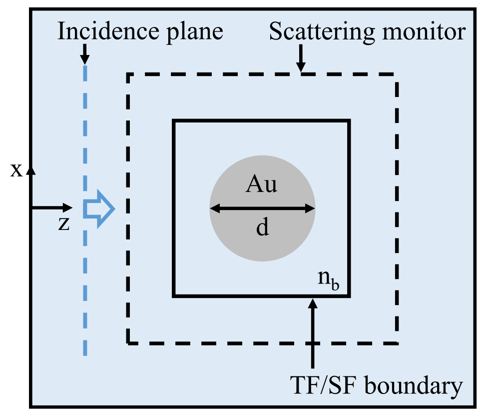

Fig. 1. Computation model.

With the TRC technique, we obtain the following equation of the DCP-FDTD method for the TM wave (the normalized expression of field components is used):

$$E^n = \frac{1-C_a}{1+C_a}E^{n-1} + \frac{D_a c\Delta t}{1+C_a}\nabla \times H^{n-\frac{1}{2}} + \frac{1}{1+C_a}\phi^{n-1} + \frac{1-C_b}{1+C_a}\eta^{n-1} \tag{2}$$

where c is the speed of light in a vacuum, and

$$\phi^n = C_c(E^n + E^{n-1}) + e^{-\nu_{\mathrm{c}}\Delta t}\phi^{n-1} + C_d(\eta^n + \eta^{n-1}) \tag{3}$$

$$\eta^n = C_a(E^n + E^{n-1}) - \phi^{n-1} + C_b\eta^{n-1}. \tag{4}$$

For the H field, the standard FDTD equation is utilized. The coefficients C_a, C_b, C_c, C_d and D_a are determined with the ratio of metal to dielectric in the cell in the DCP algorithm [4]. Inside the metal, the coefficiencts are

$$C_a = \frac{\omega_{\mathrm{p}}^2}{2\varepsilon_\infty \nu_{\mathrm{c}}}\{\Delta t - \frac{1}{\nu_{\mathrm{c}}}(1-e^{-\nu_{\mathrm{c}}\Delta t})\}$$

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$$C_b = 1$$

$$C_c = -\frac{\omega_{\mathrm{p}}^2}{2\varepsilon_\infty \nu_{\mathrm{c}}^2}(1 - e^{-\nu_{\mathrm{c}}\Delta t})^2$$

$$C_d = 0$$

$$D_a = \frac{1}{\varepsilon_\infty}$$

and the term of η is not required for updating the E field of Eq. (2).

A metallic nanocylinder is shown in Fig. 1. The metal is chosen to be Au, the relative permittivity of which is determined by Eq. (1), i.e., $\varepsilon_\infty = 9.0685$, $\omega_{\mathrm{p}}/2\pi = 2155.6$ THz and $\nu_{\mathrm{c}}/2\pi = 18.36$ THz are used [8]. The diameter of the metallic nanocylinder is $d = 50$ nm and the refractive index of the background is $n_b = 1.7$. The spatial sampling widths are determined by the number of sampling points N in the cylinder, i.e., $\Delta x = \Delta z = d/N$. The TM wave is launched from the incidence plane (see Fig.1), which consists of the super-Gaussian profile in the x direction.

Fig. 2 shows the scattering efficiency as a function of the number of sampling points in the cylinder at $\lambda = 500$ nm. Here, the scattering efficiency is evaluated using the total-field/scattered-field (TF/SF) technique [1] shown in Fig. 1. The total scattered power is obtained by summing the scattered power collected along the broken line in the SF region. As found in Fig. 2, the results of the TRC-DCP-FDTD method agree perfectly with those of the Z-transform FDTD method. The results of the DCP-FDTD method converge around $N = 50$, whereas the staircase FDTD method does not yield a converged solution even at $N = 200$.

The wavelength response of the scattering efficiency is depicted in Fig. 3. It is found that the results of the DCP-FDTD method for $N = 20$ are almost the same as those of the staircase FDTD method for $N = 200$. The staircase FDTD method results in an accuracy degradation for $N = 20$.

Here, we mention the calculation time and memory usage, where a PC with Core i7-7700k processor (4.2 GHz) is used. The TRC-DCP-FDTD method can reduce the memory usage by 10 % in comparison with the Z-transform FDTD method, with the same calculation time.

III. Conclusion

We have introduced the DCP algorithm to the FDTD method based on the TRC technique, the formulation of which is much simpler than the method with the Z-transform. To show the effectiveness of the method, we evaluate the scattering efficiency in the analysis of the metallic nanocylinder. It is shown that the DCP-FDTD method yields faster convergence in the calculation of the scattering efficiency, compared to the FDTD method with a staircase approximation. The memory usage is reduced to 90% of the DCP-FDTD method based on the Z-transform, while maintaining the same accuracy.

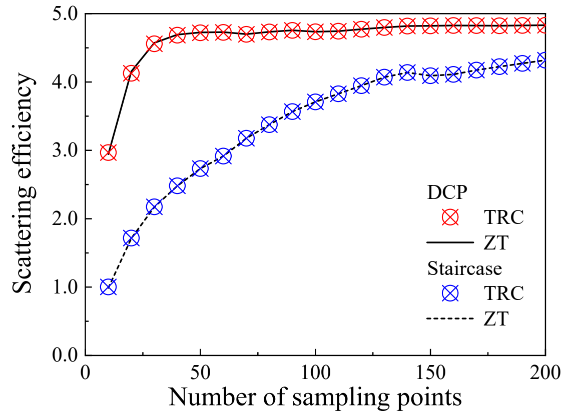

Fig. 2. Scattering efficieny versus the number of sampling points.

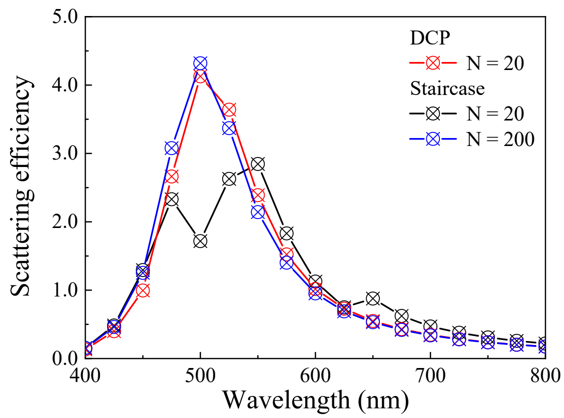

Fig. 3. Scattering efficiency versus wavelength.

References

[1] A. Taflove and S. C. Hagness, *Computational Electrodynamics: The Finite-Difference Time-Domain Method, 3rd ed.* Norwood, MA: Artech House, 2005.

[2] A. Mohammadi, H. Nadgaran, and M. Agio, "Contour-path effective permittivities for the two-dimensional finite-difference time-domain method," *Opt. Exp.,* vol. 13, no. 25, pp. 10367-10381, 2005.

[3] A. Mohammadi and M. Agio, "Dispersive contour-path finite-difference time-domain algorithm for modelling surface plasmon polaritons at flat interfaces," *Opt. Exp.,* vol. 14, no. 23, pp. 11330-11338, 2006.

[4] A. Mohammadi, T. Jalali, and M. Agio, "Dispersive contour-path algorithm for the two-dimensional finite-difference time-domain method," *Opt. Exp.,* vol. 16, no. 10, pp. 7397-7406, 2008.

[5] R. Siushansian and J. LoVetri, "A comparison of numerical techniques for modeling electromagnetic dispersive media," *IEEE Microw. Guided Wave Lett.*, vol. 5, no. 12, pp. 426-428, 1995.

[6] J. Shibayama, R. Ando, A. Nomura, J. Yamauchi, and H. Nakano, "Simple trapezoidal recursive convolution technique for the frequency-dependent FDTD analysis of a Drude-Lorentz model," *IEEE Photon. Technol. Lett.*, vol. 21, no. 2, pp. 100-102, 2009.

[7] J. Shibayama, A. Nomura, R. Ando, J. Yamauchi, and H. Nakano, "A frequency-dependent LOD-FDTD method and its application to the analyses of plasmonic waveguide devices," *IEEE J. Quantum Electron.,* vol. 46, no. 1, pp. 40-49, 2010.

[8] A. Vial, A.-S. Grimault, D. Macías, D. Barchiesi, and M. L. de la Chapelle, "Improved analytical fit of gold dispersion: Application to the modeling of extinction spectra with a finite-difference time-domain method," *Phys. Rev. B,* vol. 71, pp. 085416-1–085416-7, 2005.