Modeling Mutually Coupled Non-Identical Semiconductor Lasers on Photonic Integrated Circuits

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***Abstract*— We model the situation of two lasers in a face to face arrangement, passing through an attenuating element, where the time of flight between the lasers is on the scale typical in photonic integration (100's of μm to mm's). A modified version of the Lang-Kobayashi equations was employed to describe the interaction. By solving this delay differential equation (DDE) system we characterised different dynamical regimes including, one/two colour states, and self pulsations.**

I. Introduction

Mutually coupled lasers (MCLs) have proven a rich area of study as a system of coupled nonlinear oscillators, exhibiting interesting phenomena in the view of nonlinear dynamical systems such as multi-stabilities and coupled chaos. One interesting dynamical regime in this system is when the lasers operate in CW with their frequency and phase mutually locked (also referred to as a one-colour states), [1]. This CW regime can be used to achieve advanced modulation formats, such as Orthogonal Frequency Division Multiplexing (OFDM), [2] and Quadrature Phase Shift Keying (QPSK), [3], on a photonic integrated circuit (PIC). Other studies have theorised the existence of two-colour states in the case of identical lasers, [4], showing that the symmetry of identical lasers can be broken through mutual coupling.

We outline the existence of these one/two-colour states in the case of a frequency difference existing between the lasers along with the phenomena of self pulsations. We aim to present these dynamics in such a way that they are experimentally verifiable (i.e. how they would manifest on specific pieces of equipment), for comparison with experimental work from our group.

II. Model

The model used in this work is a modified version of the Lang-Kobayashi equations for single mode lasers under feedback, and can be seen in (1).

$$
\begin{aligned}
E_1'(t) &= i\delta E_1 + (i+i\alpha)E_1 N_1 + \eta e^{-i\phi} E_2(t-\tau)\\
E_2'(t) &= (i+i\alpha)E_2 N_2 + \eta e^{-i\phi} E_1(t-\tau)\\
N_1'(t) &= \varepsilon[J - N_1 - (N_1+\nu)|E_1|^2]\\
N_2'(t) &= \varepsilon[J - N_2 - (N_2+\nu)|E_2|^2]
\end{aligned}
\tag{1}
$$

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These equations describe the temporal evolution of the complex electric fields, $E_{1,2}$, and excess carrier densities, $N_{1,2}$.

α	Linewidth enhancement factor	2.6
J	Injection parameter	0.23
ε	Ratio of carrier and photon lifetimes	0.0025
η	Coupling coefficient	Variable
ϕ	Coupling phase	Variable
δ	Frequency detuning of the lasers	Variable
τ	Delay time between the lasers	Variable

The complex electric fields are then decomposed into a real and imaginary part such that $E_{1,2} = E_{x_1} + iE_{y_1}$ giving six coupled Delay Differential Equations (DDEs).

DDEs require a history of the variables, i.e. when $t < \tau$. In the time interval $0 \leqslant t \leqslant \tau$ the lasers are uncoupled as the light is "in flight" between the lasers, therefore the history comes from modeling two uncoupled lasers. This system equations can be solved using Mathematica's NDSolve.

III. One/two-colour states

One and two-colour states are when the complex electric fields take the following forms respectively.

$$E_1 = a_1 e^{i\omega_a t}, \qquad E_2 = a_2 e^{i\omega_a t} e^{i\delta_a} \tag{2}$$

$$E_1 = a_1 e^{i\omega_a t} + b_1 e^{i\omega_b t}, \quad E_2 = a_2 e^{i\omega_a t} e^{i\delta_a} + b_2 e^{i\omega_b t} e^{i\delta_b} \tag{3}$$

A. Simulation results

To observe these phenomena theoretically the Fast Fourier Transform (FFT) of $E_{1,2} = E_{x1,2} + iE_{y1,2}$ is taken. One-colour states display one clear frequency peak and two colour states have two clear peaks. The physical devices can be seen in Fig.1. The attenuating section between the lasers is referred to as a Variable Optical Attenuator (VOA), this is a short waveguide section, that can be independently biased. The material used in fabricating the device is a PIN structure, and by reverse biasing this junction the material becomes absorbing, decreasing the fraction of light reaching one laser from the other, analogous to the coupling coefficient, η, in the theoretical model.

In experiments, the lasers are arranged to have a specified frequency difference, δ, and the reverse bias of the VOA is swept to slowly introduce the lasers. This was re-created theoretically by, sweeping the η parameter, and at each η, taking the FFT of the complex fields, stacking each graph, and viewing it in a contour plot, as seen in Fig.2a. The two-colour states can be seen on the left hand side of the diagram, where there are two lines diverging. At approximately $\eta = 0.3$ the system changes to a one-colour state.

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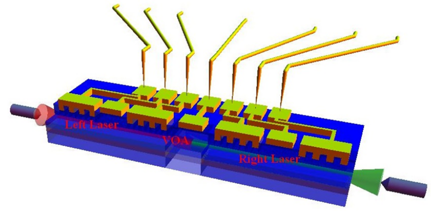
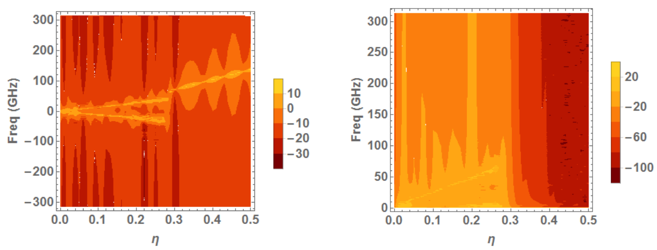


Fig. 1: Graphical representation of MCL device



(a) FFT of E (b) FFT of $|E|^2$

Fig. 2: Logarithmic contour plots of $E/|E|^2$ for varying coupling coefficient, for δ=0.01, $\tau = 0.5$, and $\phi = 0.4\pi$

The variables δ and τ are scaled by the photon lifetime, τ_p, of the laser. This lifetime can be calculated using (4).

$$\tau_p = \frac{-2L}{cln[R_1R_2(1-T_i)^2]} \tag{4}$$

Here, L is the cavity length, c is the speed of light, T_i is the transmission, and $R_{1,2}$ are the mirror reflectivities. For the semiconductor lasers used $\tau_p \approx 10ps$, meaning for a detuning of 1GHz, $\delta = 0.01$, and $\tau = 1$ translates to a separation $\approx 1mm$. To observe these dynamics experimentally an Electronic Spectrum Analyser (ESA) would be used. This shows the Fourier transform of a voltage signal received by a photodiode. The photodiode measures $|E|^2$, therefore the ESA displays the Fourier transform of $|E|^2$. Repeating the process for obtaining Fig.2a, but this time showing the FFT of $|E|^2$ yields Fig.2b. Now the two-colour state manifests as a single peak, whose frequency increases as the peaks of the two-colour state diverge in Fig.2a. Taking the absolute value squared of the definition of a two-colour state, (3), gives:

$$|E_1(t)|^2 = a_1^2 + b_1^2 + 2a_1b_1Cos((\omega_a - \omega_b)t) \tag{5}$$

Fourier transforming this would give a single beat note, between the frequencies $\omega_{a,b}$, as observed. Similarly for the one colour states $|E_1|^1 = a_1^2$, a constant, yielding an FFT with no distinct frequencies, as seen on the right of Fig.2b.

IV. Self-Pulsations

In [1], when detuning was included (but not delay), a regime of self-pulsations was discovered, by setting $\phi = \frac{\pi}{2}$. These are periodic oscillations in the field intensity $|E|$. By making the same simplification to the full DDE model, the self-pulsations still remain, as can be seen in Fig.3.

A repeat of the procedure in Section III, sweeping η and taking the FFT of $|E|^2$ gives Fig.4. This is identical to the Fig.2b, for the two colour state. Hence it can be said that two colour states cause self-pulsations within the laser.

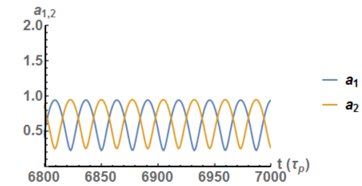


Fig. 3: Time traces of $|E_{1,2}| = a_{1,2}$ for for δ=0.1, $\tau = 0.5$, and $\eta = 0.1$

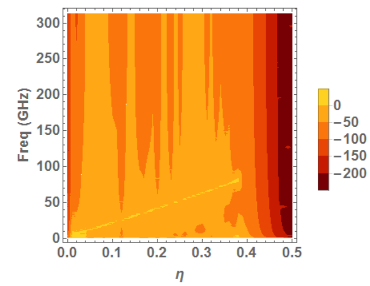
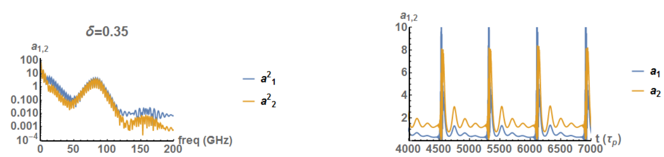


Fig. 4: FFT of $|E_1|^2$ logarithmic contour plot for varying coupling coefficient, for the DDE model with $\phi = \frac{\pi}{2}$ simplification. δ=0.01, $\tau = 0.5$

However more complicated self-pulsations exist, which cannot be caused by a two-colour state. For example in Fig.5b, a more complex oscillation exists resulting in a FFT of the $|E|^2$ (Fig. 5a), differing from that of a two colour state. We are looking to match these more complicated pulsations with those seen in experiment.



(a) FFT of $|E|^2$ (b) Time trace of $|E|^2$

Fig. 5: Self pulsations for $\delta = 0.35$, $\eta = 0.4$ and $\tau = 1.5$

V. Conclusions

This work has shown the existence of one/two-colour states for the full DDE system for MCLs with detuning, and how a two colour states causes self-pulsations in both lasers. Work is ongoing to analytically verify these results.

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