Bifurcation and frequency analysis of mutually delay-coupled semiconductor lasers in photonic integrated circuits

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***Abstract*—A system of two mutually delay-coupled semiconductor lasers for integration in a photonic integrated circuit is investigated. Multi-stabilities and bifurcation scenarios are presented, followed by a comprehensive frequency analysis of the symmetric and symmetry-broken, 1-colour and 2-colour states.**

I. Introduction

We theoretically investigate the dynamics of two mutually delay-coupled semiconductor lasers in a face to face configuration for integration in a Photonic Integrated Circuit (PIC). We consider lasers separated by a distance d which are coherently coupled via their optical fields. The time delay, $\tau = d/c$, where c is the speed of light, arises from the finite propagation time of the light from one laser to the other one. This system is of importance in the creation of advanced modulation formats within a PIC, and has been widely studied, both experimentally and theoretically [1], [2]. The system is well described by Lang Kobayashi rate equations, which are a system of delay differential equations (DDEs) with one fixed delay. Yanchuk et al. [3] studied this system in the limit of zero delay and predicted one-colour symmetric states for very small delays. Later Erzgräber et al. [4] studied the bifurcations of 1-colour states for large delays. Moreover, for zero delay, stable symmetric and symmetry-broken 1-colour and 2-colour states have been recently predicted by Clerkin et. al. [5]. In particular, symmetry-broken 2-colour states are highly interesting from an application point of view, for example in the context of all-optical switching [5]. Here we show that these states continue to exist for finite delays. A detailed study of the relevant bifurcations of the system with finite delays in the range of τ=0-1 (in units of photon lifetime) was performed, using the continuation Matlab package DDE-BIFTOOL.

II. Rate equation model

The Lang-Kobayashi-type rate equation has been used to model the system of two mutually coupled lasers as [4]:

$$\frac{\mathrm{d}E_1(t)}{\mathrm{d}t} = (1+i\alpha)N_1(t)E_1(t)+\kappa e^{-iC_p}E_2(t-\tau)-i\Delta_1, \quad (1)$$

$$\frac{\mathrm{d}E_2(t)}{\mathrm{d}t} = (1+i\alpha)N_2(t)E_2(t)+\kappa e^{-iC_p}E_1(t-\tau)-i\Delta_2, \quad (2)$$

$$T\frac{\mathrm{d}N_1(t)}{\mathrm{d}t} = P - N_1(t) - (1+2N_1(t))|E_1(t)|^2, \quad (3)$$

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$$T\frac{\mathrm{d}N_2(t)}{\mathrm{d}t} = P - N_2(t) - (1+2N_2(t))|E_2(t)|^2, \quad (4)$$

Here E_1 and E_2 are the normalized complex slowly-varying envelope of the optical fields, and N_1 and N_2 are the normalized inversions for laser 1 and laser 2, respectively. Also, T, α and P are parameters of the individual lasers. $T = 392$ is the normalized carrier lifetime (in units of the photon lifetime). $\alpha = 2.6$ is the linewidth enhancement factor, and $P = 0.23$ is the pumping parameters, which describes the amount of the electrical current used to pump the electron-hole pairs in each lasers. Time t is measured in the units of photon lifetime, τ_p, which we estimate to be around 7.7 ps.

The main bifurcation parameters are the coupling phase C_p and coupling rate κ. As we are interested in modelelling lasers within a PIC, we consider a separation of around 1.5 mm between lasers, which translates into a value of τ between 0 and 1. In this paper we concentrate on the case with zero detuning $\Delta = 0$.

Equations (1)-(4) are a system of DDEs with a single fixed delay. The dynamical variables E_1 and E_2 are slowly varying quantities relative to a central frequency Ω_0. Therefore the optical fields of laser 1 and 2 are given by $E_1(t)\exp(i\Omega_0 t)$ and $E_2(t)\exp(i\Omega_0 t)$. In this work we have performed a detailed study of the relevant bifurcations of the system using the MATLAB package DDE-BIFTOOL [6], which reveals the behaviour of the mutually coupled lasers, and predicts the stability regions for wide range of parameters [7]. We also solve the rate equations (1)-(4) for given τ, κ and C_p numerically.The calculated optical field frequencies indicate the existence of one-colour and two-colour, symmetric and symmetry-broken states for different values of parameters.

III. Results and discussion

A simple one-colour ansatz for the solution of system (1)-(4) is given by $E_1(t) = R_1^s e^{i\omega_s t}$, $E_2(t) = R_2^s e^{i\omega_s t + i\sigma}$, $N_1(t) = N_1^s$ and $N_2(t) = N_2^s$, where $R_{1,2}^s$, $N_{1,2}^s$, ω_s, and σ are suitable real constants. In the case of $R_1 = R_2$ we have a symmetric one-colour state. In this case, the constant ω_s, which is the deviation of the frequency of the coupled laser system from solitary laser frequency Ω_0, is given by

$$\omega_s = \pm\kappa\sqrt{1+\alpha^2}\sin(C_p + \omega_s\tau + \arctan(\alpha)) \quad (5)$$

Here $+$ and $-$ correspond to in-phase ($\sigma = 0$) and anti-phase ($\sigma = \pi$) solutions, respectively. We use this symmetric one-colour solution as a starting point for the numerical continuation and explore the bifurcations, which occur as the

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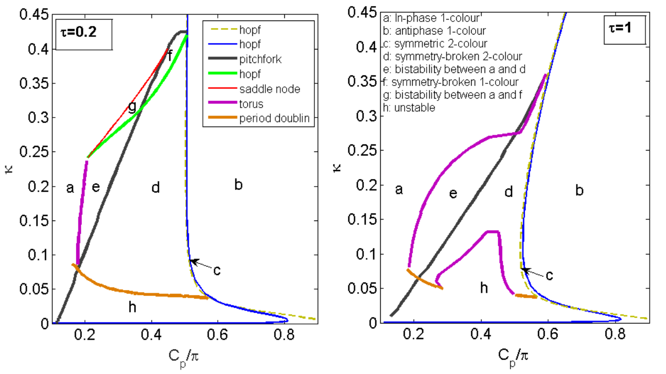

Fig. 1. Pitchfork (solid lines) and Hopf (dashed lines) bifurcation diagram of system (1)-(4) for $\tau = 0.01 - 2$.

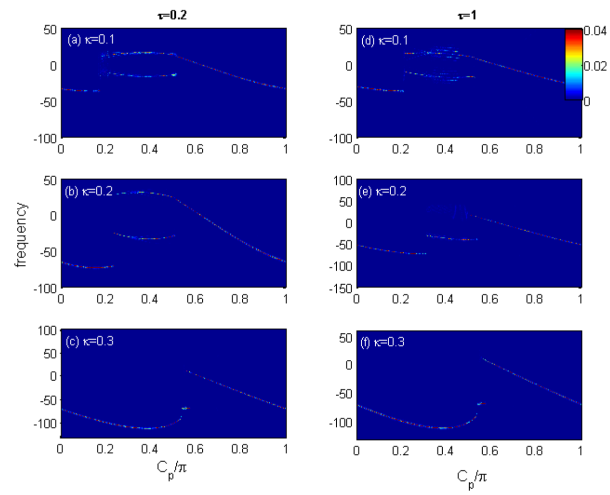

Fig. 2. The frequency spectrum, relative to the frequency of the free-running lasers, versus the coupling phase C_p for $\tau = 0.2$ and 1, for coupling strength $\kappa = 0.1$ (a and d), $\kappa = 0.2$ (b and e) and $\kappa = 0.3$ (c and f).

two bifurcation parameters coupling phase C_p and coupling strength κ are varied. Fig. 1 shows the bifurcation diagrams for $\tau = 0.2$ (left panel) and $\tau = 1$ (right panel). Hopf, pitchfork, saddle node, torus and period doubling bifurcations are encountered as indicated by the various bifurcation lines, which separate the parameter space into eight ($\tau = 0.2$) or six ($\tau = 1$) distinct regions.

Regions a and b presents stable in-phase and antiphase one-colour states. This is consistent with one-colour locked regions in Fig 1. In region c, which is a tiny area between the two Hopf lines, it is possible to find stable symmetric two-colour states. Region d of Fig. 1 shows existence of stable symmetry-broken two-colour states for intermediate coupling rate and phase. For small delay times, there is a region (region f in left hand panel of Fig. 1) with a stable symmetry-broken one-colour state. However, this region vanishes for larger delay times (right hand panel). Region e represents region of bistability between one-colour symmetric or two-colour symmetry-broken states, and it depends on the initial condition, which of the two states is obtained. Similarly region g shows a bistability between symmetric one-colour and symetry-broken one-colour states [7].

We have also performed a comprehensive frequency analysis by solving DDEs in Eqs. (1)-(4) numerically using an explicit Runge-Kutta algorithm. The optical frequency of the lasers is calculated via a Fourier transform of the optical field of the lasers. In Fig. 2 the calculated optical frequency of the lasers are shown for $\tau = 0.2$ and 1, for 3 different values of coupling $\kappa = 0.1$, 0.2 and 0.3. We have found that for small and high values of phase ($C_p < 0.2$ and $C_p > 0.6$) the lasers are locked to a single common frequency. The frequency of the symmetric one-colour state, can be also calculated analytically, using Eq. 5 which shows a perfect match with numerical results.

Moreover, in agreement with the result of Fig. 1, stable symmetry-broken two-colour states are observed for intermediate coupling, where both lasers lase simultaneously at two optical frequencies which are separated by up to 150 GHz. We have also calculated the frequency of symmetry-broken two-colour states, using an analytical approach which gives a very good agreement with the result of Fig. 2.

IV. Conclusion

We have shown that the stable symmetry-broken two-colour states continue to exist for finite delay time, and therefore stable states exist for almost all values of the coupling phase C_p and delay time up to 1. However, increasing delay time makes the area with intermediate C_p and low value of κ unstable. We observe that the frequency of one colour states changes linearly with coupling κ. We are currently trying to benchmark our results against the experiments that are carried out in our group.

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