

# Coupled Wave Analysis of Higher Order Laterally Coupled Silicon Nitride Bragg Gratings

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**Abstract**—In this work we compare numerical simulations of silicon nitride waveguide Bragg gratings based on a 2D+z coupled wave theory implementation against fabricated and measured structures. The coupled wave theory presented here includes additional coupling terms to account for radiative losses inherent to higher order gratings. In conclusion, the presented theory reflects the measured spectra and is therefore an efficient tool to design waveguide Bragg gratings.

**Keywords**—coupled wave theory, Bragg grating, silicon nitride, waveguide

## I. INTRODUCTION

Integrated photonic devices operating in the visible to near infrared optical wavelength region provide a versatile platform for sensing applications such as medical diagnostics, environmental monitoring or food safety [1]. Using low-temperature plasma enhanced chemical vapor deposition (PECVD) to grow silicon nitride (SiN) layers enables the co-integration of optical waveguides together with silicon photodiodes and CMOS circuitry. Among various photonic building blocks for integrated optical waveguide devices, Bragg gratings represent a versatile component. They can act as passive mirrors, be part of resonators and filters, or function as distributed feedback (DFB) structures for laser sources in the presence of optical gain. In this work, we implement the coupled wave theory (CWT) in 2D+z including additional coupling terms from Streifer [2] to account for higher order partial modes occurring in higher order Bragg gratings. These partial modes contribute to both coupling and radiative losses. Numerical results are benchmarked against measurements of 3<sup>rd</sup> order silicon nitride Bragg gratings.

## II. THEORY AND MODELLING

### A. Coupled wave theory

The periodic modulation of the material permittivity  $\epsilon_r$  of the grating can be expanded into a Fourier series along the propagation direction  $z$  with grating period  $\Lambda$

$$\epsilon_r(x, y, z) = A_0(x, y) + \sum_{\substack{q=-\infty \\ q \neq 0}}^{\infty} A_q(x, y) e^{2\pi i q z / \Lambda}, \quad (1)$$

which is directly integrated into the wave curl-curl equation. The homogeneous part of the resulting differential equation is used to obtain the mode solution for the electric field  $\vec{E}_0$ . The coupling coefficient  $\kappa_p$  is given by [2, 3]

$$\kappa_p = \frac{k_0}{2\beta_0 P} \iint_G A_p(x, y) |\vec{E}_0|^2 dx dy, \quad (2)$$

where  $P$  is the power of the fundamental mode and  $p$  the negative grating order.

However, additional coupling and losses arise from higher order partial waves  $\vec{E}_m^{(i)}$  derived from inhomogeneous differential equations [2]. The contribution of these waves are introduced into the coupled wave equations via the coefficients

$$\begin{aligned} \zeta_1 &= \sum_{\substack{q=-\infty \\ q \neq 0, -p}}^{\infty} \eta_{q, -q}^{(0)} & \zeta_2 &= \sum_{\substack{q=-\infty \\ q \neq 0, -p}}^{\infty} \eta_{q, -q}^{(p)} \\ \zeta_3 &= \sum_{\substack{q=-\infty \\ q \neq 0, p}}^{\infty} \eta_{q, p-q}^{(p)} & \zeta_4 &= \sum_{\substack{q=-\infty \\ q \neq 0, p}}^{\infty} \eta_{q, p-q}^{(0)} \end{aligned} \quad (3)$$

with

$$\eta_{r,s}^{(i)} = \frac{k_0^2}{2\beta_0 P} \iint_G A_r \vec{E}_0^* \vec{E}_s^{(i)} dx dy. \quad (4)$$

The resulting coupled wave equations, which couple the forward-propagating field  $R(z)$  with the backward-propagating field  $S(z)$ , have the modified form

$$\frac{dR}{dz} + (-i\delta - i\zeta_1)R = i(\kappa_p^* + \zeta_2)S, \quad (5)$$

$$\frac{dS}{dz} + (-i\delta - i\zeta_3)S = i(\kappa_p + \zeta_4)R. \quad (6)$$

Coefficients for power reflection and transmission coefficients are obtained by suitable boundary conditions of the coupled equations.

### B. Modelling of the grating region

A third order Bragg grating designed for a resonance wavelength of 850 nm in TM-like polarization is used for the

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benchmarking of the reflectivity and transmission values between CWT and measurement. As the Fourier expansion in (1) facilitates the modelling of complex shapes, deviations between initial design and fabricated structures can be accounted for in simulations. For that purpose, a sample without cladding material but otherwise identical fabrication steps has been measured with scanning electron microscopy (SEM) and the Fourier coefficients  $A_q(x,y)$  have been derived thereof. Figure 1 shows a section of the fabricated laterally coupled grating obtained by SEM. Table 1 summarizes the geometry parameters and material constants used in the simulation.

TABLE I. SIMULATION PARAMETERS

Geometry				Material	
Width	Height	Period	Length	$n_g$ (SiN)	$n_c$ (SiO <sub>2</sub> )
1046 nm	160 nm	844 nm	250 μm	1.9107	1.4576

III. RESULTS

Reflection and transmission spectra obtained by CWT and optical measurements are shown in Fig. 2. The resonance wavelength for the TE-like mode is shifted from 848 nm to 874 nm as the effective index of the TE-like mode is higher. For both polarizations the simulation predicts a significant radiative loss of 20 dB (TE) and 6 dB (TM) respectively. This has been verified by the measurement. The increased loss and coupling results from the stronger confinement of the TE-like mode. The coupling coefficient  $\kappa_p$  has been overestimated by the CWT simulation for both polarizations, resulting in a pronounced dip in the transmission spectrum for the TE-like mode. As the coupling coefficients  $\kappa_p$  and  $\zeta_i$  directly depend on the Fourier series representation of the grating, small deviations between model and fabricated structures have a significant influence on the spectra. Even small measurement uncertainties of 1% of the waveguide width lead to a translation of the spectra of about 1 dB in the case of TM polarization and presented geometries.

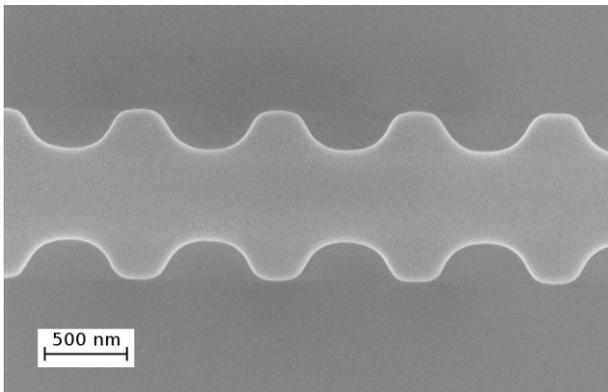


Fig. 1. SEM micrograph of analyzed silicon nitride laterally coupled waveguide Bragg grating. Single side modulation depth is 244 nm, total width 1046 nm, and grating period is 844 nm.

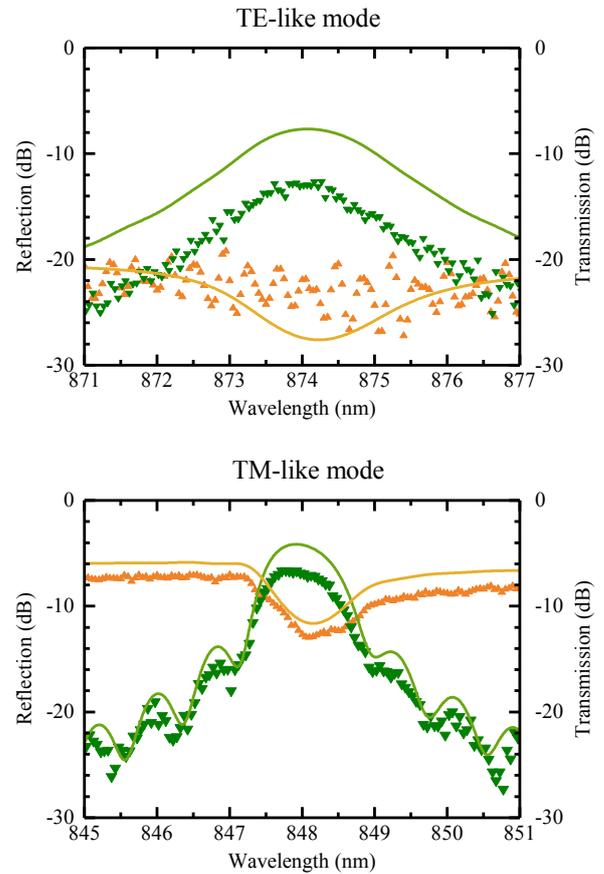


Fig. 2. Reflection and transmission spectra for TE and TM-like mode for 3<sup>rd</sup> order laterally coupled Bragg grating. Dimensions for CWT simulations are calculated from SEM micrograph. (▲ trans. measured, ▼ refl. measured, — trans. CWT, — refl. CWT)

IV. CONCLUSION

Coupled wave theory including higher order partial waves has been used to obtain reflection and transmission spectra for a 3<sup>rd</sup> order laterally coupled silicon nitride Bragg grating for both polarizations. The simulation is able to predict radiative losses inherent to higher order Bragg gratings. The Fourier coefficients used to model the grating have been extracted from a SEM micrograph to account for deviations from design.

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