

# Gain and Linewidth Enhancement in Quantum Dots with External Electric Field

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**Abstract**—Material gain and linewidth of Quantum Dot ensemble are calculated assuming the Gaussian distribution of the density of states due to the size-deviation of dots. The effect of electric field is incorporated in the analysis through the mean and variance of energy states. The results showing the enhancement of optical gain and linewidth with electric field indicate important applications in sub-cellular medical imaging.

**Keywords-component:** Quantum Dot; Gain; Inhomogeneous broadening; Electric field; Linewidth

## I. INTRODUCTION

The complete confinement of carriers plays key role in the novelty of the performance of quantum dot (QD) based photonic devices. As already reported in literature, QD LED offers low turn-on voltage and high quantum efficiency [1], QD solar cell offers high power conversion efficiency [2], QD SLEDs find application in optical coherence tomography (OCT) [3]. QDs grown on a layer, such as using Stranski - Krastanov growth method, vary in size, leading to the density of states (DOS) to deviate from the ideal delta function. This inhomogeneous broadening results in increase in spectral linewidth. High gain and linewidth of optical sources are important for OCTs in sub-cellular imaging. As an external electric field affects the energy states [4], it is expected to change the DOS and other dependent optical properties. In this paper, we investigate the effects of the external electric field on the gain and linewidth of QD ensemble.

## II. THEORY

When an external electric field,  $E_f$  is applied along  $z$ -direction in QD, the  $z$ -dependent wave function ( $Z$ ) is given by,

$$-\frac{\hbar^2}{2m^*} \frac{\partial^2 Z}{\partial z^2} - eE_f z Z = E_{zn} Z \quad (1)$$

where  $E_{zn}$  is energy eigen-value of a single QD of dimensions  $d_1$ ,  $d_2$ , and  $d_3$  along  $x$ ,  $y$ , and  $z$ -directions respectively, and  $n=1,2,3, \dots$  is quantum number along  $z$ -direction. The solution of (1) is a combination of Airy functions [4]. Using the appropriate boundary conditions, the wave functions and energy eigen-values are calculated numerically. The density of

states (DOS) of an ensemble of QDs having surface dot density,  $N_D$  is assumed to have Gaussian distribution and is given by the following expression:

$$B_0(E - \bar{E}_0) = \frac{N_D}{d_3} \frac{1}{\sqrt{2\pi}\sigma_0} \exp\left[-(E - \bar{E}_0)^2 / 2\sigma_0^2\right] \quad (2)$$

where  $\bar{E}_0$  is the mean energy and  $\sigma_0$  is the standard deviation in absence of electric field.

As mentioned earlier, the eigen-energy becomes a function of electric field, so the mean energy in (2) changes ( $\bar{E}$ ). An approximate expression for  $\sigma$  in presence of electric field is obtained as follows:

$$\sigma = \sigma_0 e^{((\bar{E} - \bar{E}_0)^2 / 2\sigma_0^2)} \quad (3)$$

where field dependence appears through  $\bar{E}$ .

Next, the absorption and emission coefficients are calculated using the Fermi's golden rule. The material gain of the QD ensemble is given by the difference between the emission and absorption coefficients [5]:

$$g(E) = \frac{2\pi e^2}{cn_r \epsilon_0 m_0^2} \sum_{f,i} \frac{|P_{fi}|^2}{\omega_{fi}} [f_f(E) - f_i(E)] B_0(E - E_{fi}) \quad (4)$$

where  $i$  is the initial state,  $f$  is the final state,  $P_{fi}$  is the momentum matrix element (containing wave functions overlapping term),  $\omega_{fi}$  is the frequency of the incident light,  $n_r$  is the refractive index of the QD material,  $c$  is free space light velocity,  $m_0$  is the free electron mass,  $\epsilon_0$  is free space permittivity,  $e$  is electronic charge,  $f_i(E)$  and  $f_f(E)$  are Fermi occupation probability of electron in valence band and conduction band, respectively.

For the calculation of gain, quasi Fermi levels are obtained for given carrier concentrations. The relation between carrier concentration and quasi Fermi level are derived as follows:

$$n = \frac{N_D}{d_3} \operatorname{erfc}\left(\frac{\sigma^2/KT}{\sqrt{2\pi}\sigma}\right) \sum_{E_{lmn}} \left[ \exp\left(\frac{E_{fn} - E_{lmn}}{KT}\right) + \frac{\sigma^2}{(KT)^2} \right] \quad (5)$$

$$p = \frac{N_D}{d_3} \operatorname{erfc}\left(\frac{\sigma^2/KT}{\sqrt{2\pi}\sigma}\right) \sum_{E_{l'm'n'}} \left[ \exp\left(\frac{E_{fp} - E_{l'm'n'}}{KT}\right) + \frac{\sigma^2}{(KT)^2} \right] \quad (6)$$

III. RESULTS

The results shown below are done for InAs/ GaAs QD system as an example with dot size of 10 nm. Material gain is computed using (4) and plotted for different electric fields in Fig. 1. A red shift in the gain peak is observed because of the lowering of the quantized energy state of QD. It may also be noted that the peak gain increases with the electric field. The variation of the peak gain with electric field is shown in Fig. 2.

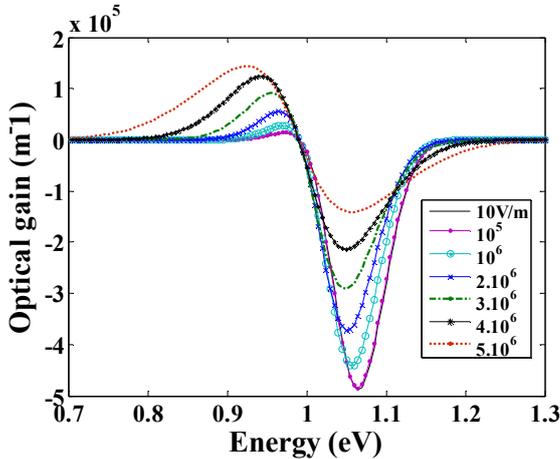


Figure 1. Optical gain spectrum for different electric fields

The increase in gain may be due to the reason that the carrier occupancy of the quantized energy states is large as electric field increases. So, the upward optical transition for absorption decreases while the downward transition for emission increases, giving rise to gain enhancement.

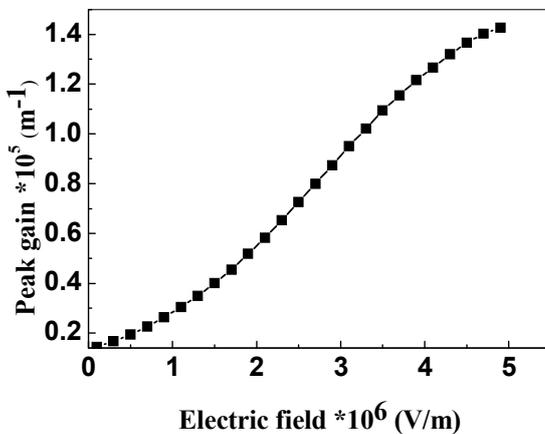


Figure 2. Variation of peak gain with electric field

Fig. 3 shows the linewidth enhancement with electric fields. The enhancement is small for low field. But there is rapid enhancement for high electric field. The enhancement is due to the change in standard deviation with the external electric field.

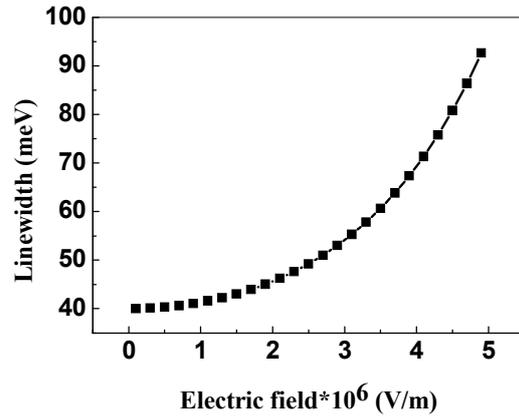


Figure 3. Variation of linewidth with electric field

IV. CONCLUSIONS

The results show that there is enhancement in both red-shifted optical gain and linewidth of the quantum dot ensemble, with the application of external electric field. Thus, external electric field can be used to achieve the high gain and broadband requirement of optical sources in optical coherence tomography for sub-cellular imaging.

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