

# Numerical Simulation of Waveguide Light Scattering for Si Photonics

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**Abstract**—We developed 3D-fullwave S-matrix analyzer. The S-matrix can shows details of the scattered-waves from optical sturcture in frequency domain.

## I. INTRODUCTION

A silicon optical waveguide has a large refractive index difference, and a precise three-dimensional full-wave model is required for numerical analysis. Light scattering to be analyzed is not only the propagation loss of the main mode. There are a wide variety of analysis such as transmittance and reflection of silicon optical modulator [1], phase fluctuation in optical circuits using interference [2], reflection in waveguides of silicon-photonics devices mounting a semiconductor laser [3], and inter-mode scattering in multi-mode waveguides used to suppress loss.

## II. FREQUENCY DOMAIN SIMULATION

To develop a frequency domain simulator [4], we consider the Maxwell equations in  $\omega$  space:

$$\begin{aligned} \nabla \times \mathbf{H} &= -i\omega\epsilon_0\epsilon(\mathbf{x})\mathbf{E}, \\ \nabla \times \mathbf{E} &= i\omega\mu_0\mu(\mathbf{x})\mathbf{H}, \end{aligned} \quad (1)$$

where  $\epsilon_0$  ( $\mu_0$ ) is vacuum permittivity (permeability). The symbol  $i$  denotes an imaginary number, and the notation “ $\exp(-i\omega t)$ ” describes a harmonic oscillation.

First, we calculated propagation modes [5] for a silicon optical waveguide as shown in Fig. 1(a) There are three waveguide modes  $TE_0$ ,  $TM_0$  and  $TE_1$  (see Fig. 1(b)). Other modes are radiation modes in the clad region, and note that these have almost no energy flow in the waveguide.

Next, we calculate a scattering matrix (S-matrix) by using the discretized equations from eq. (1):

$$\begin{aligned} \Delta_z \begin{pmatrix} \mathbf{H}_x \\ \mathbf{H}_y \end{pmatrix} &= i\mathbf{M}_{HE} \begin{pmatrix} -\mathbf{E}_y \\ \mathbf{E}_x \end{pmatrix}, \\ -\Delta_z^T \begin{pmatrix} -\mathbf{E}_y \\ \mathbf{E}_x \end{pmatrix} &= i\mathbf{M}_{EH} \begin{pmatrix} \mathbf{H}_x \\ \mathbf{H}_y \end{pmatrix}. \end{aligned} \quad (2)$$

$\Delta_z$  is the forward difference operator for  $z$ -axis, and  $\mathbf{M}_{HE}$  and  $\mathbf{M}_{EH}$  are transfer operators between electric field  $x, y$  elements and magnetic field elements as shown in Fig. 2. Numerical algorithm for scattering problem of quantum-wave [6] can stably calculate the S-matrix for eq. (2).

We show numerical results of scattering for sidewall grating structure in Fig. 3 (a). Figure 3 (b) shows

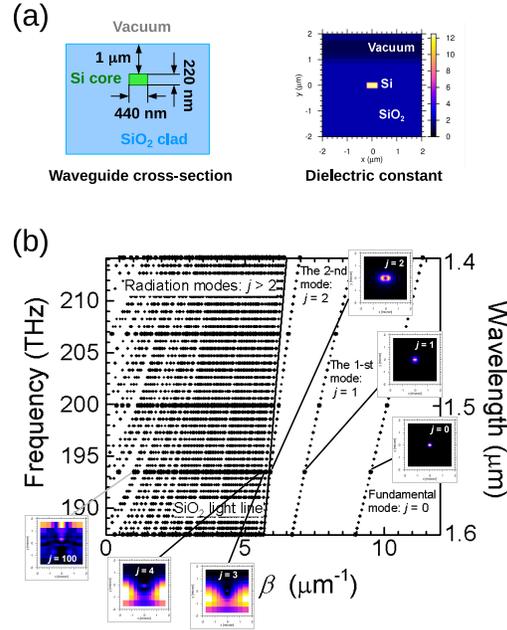


Fig. 1. (a) Cross-section of Silicon optical waveguide. (b) Dispersion of propagation modes.

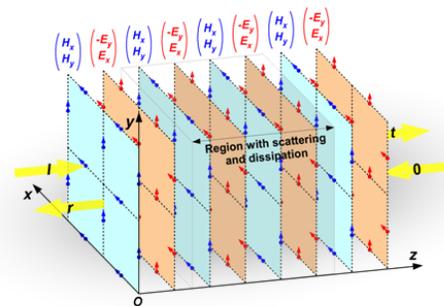


Fig. 2. Electromagnetic fields in Yee lattice can be stably connected from left side to right side.

that transmittance of  $TE_0$  mode becomes almost 1 at  $1.55\mu\text{m}$ , although the back-scattering element in  $TE_0$  mode has a stop band at  $1.45\mu\text{m}$ . This simulation result was useful for the design of the optical modulator [1].

## III. CURVILINEAR COORDINATES WITH TWO CURVATURES

For application to various optical structures, we try to apply orthogonal curvilinear coordinates ( $u_0, u_1, u_2$ ) with center line of waveguide as  $u_2$  axis to the simulator (see fig. 4(a)). The curvilinear coordinates are defined by the curvature  $\kappa_b$  representing the waveguide bending and the curvature  $\kappa_w$  representing the increase

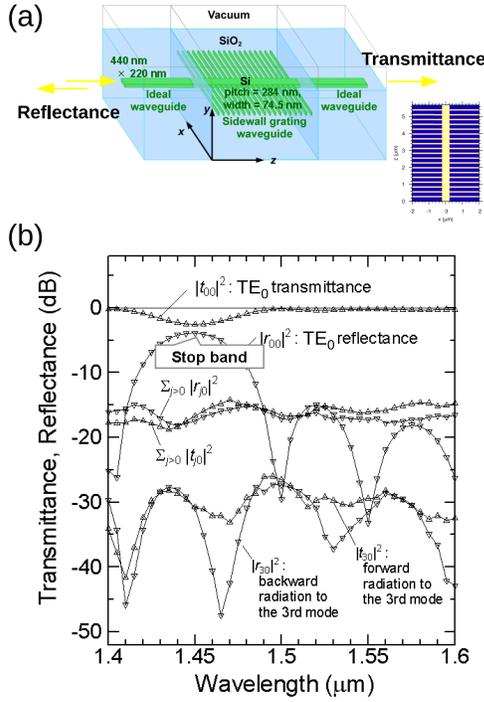


Fig. 3. (a) Sidewall grating waveguide. (b) S-matrix elements for scattering by the sidewall grating.

or decrease in width (taper). In the  $(u_0, u_1, u_2)$  space, we consider only the influence of  $\kappa_b$  and  $\kappa_w$  without noting shape of the waveguide. Furthermore, Line-Width Roughness (LWR)  $\Delta_W$  and Line-Center Roughness (LCR)  $\Delta_C$  caused by semiconductor process are linked to  $\kappa_b$  and  $\kappa_w$ :

$$\kappa_w(u_2) \approx \frac{1}{\langle W \rangle} \frac{d\Delta_W}{du_2} \quad \text{and} \quad \kappa_b(u_2) \approx \frac{d^2\Delta_C}{du_2^2},$$

where  $\Delta_W$  and  $\Delta_C$  are defined by two Line-Edge Roughness (LER)  $\Delta_1$  and  $\Delta_2$  (see fig. 5):

$$\Delta_W(u_2) = \Delta_1(u_2) - \Delta_2(u_2)$$

and

$$\Delta_C(u_2) = \frac{\Delta_1(u_2) + \Delta_2(u_2)}{2}.$$

We can evaluate TE<sub>0</sub>-mode scattering due to LWR and LCR in the framework of the orthogonal curvilinear coordinates. Our preliminary analysis obtains that back scattering becomes dominant in the scattering process. We will show details of our approach and differences from other theories [7], [8].

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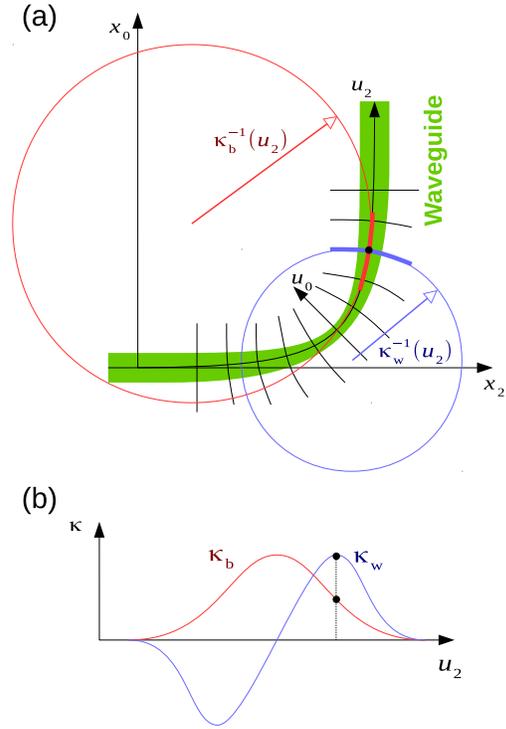


Fig. 4. (a) Curvatures  $\kappa_b$  and  $\kappa_w$  for waveguide. (b) The curvatures are given as functions of propagation axis  $u_2$ .

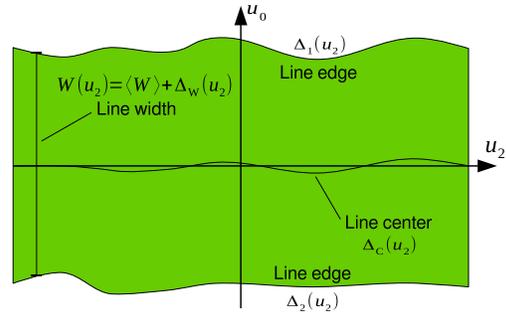


Fig. 5. Waveguide roughness.  $\Delta_W$  and  $\Delta_C$  are line width roughness and line center, respectively.

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