

Sub-wavelength air cavities in uniaxial metamaterials

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Abstract- Reducing the size of device components, e.g. waveguides and resonant cavities, beyond the diffraction limit is a means to an end. Here we demonstrate the condition required to have air cavities within a uniaxial metamaterial clad waveguide. Our work reveals that air cavity sizes much smaller than the operating wavelength ($D^2h/\lambda^3=6.7e-4$) are achievable under specific conditions of cladding material parameters, which could have great impact on miniaturization of electromagnetic devices.

I. INTRODUCTION

Recent advances in metamaterials, man-made artificial composite materials with structures much smaller than operating wavelength, have demonstrated the possibility to overcome the diffraction limit and open opportunities to create a new generation of compact devices [1]. In 2002, Engheta proposed the idea of using metamaterials to create sub-wavelength cavity resonators [2]. He demonstrated that the dimension of a 1D cavity resonator could be reduced when the dielectric medium in between two perfect metallic plates is partially replaced with a lossless negative index metamaterial layer. The metamaterial layer acts as a phase compensator for the dielectric layer. In practice, with most metamaterials relying on metallic structures, such cavities have relatively high material losses.

Caiazzo et al. expanded the concept to microwave air cavities [3], by inserting frequency selective surfaces (metamaterial surfaces) inside a microwave air cavity leading to size reduction of 1/7. In another approach, Li et al. implemented metamaterial-based air cavities by replacing the conventional metallic reflectors of the cavities with metamaterial reflectors, which reflect waves with a definite phase allowing the opportunity to reduce the cavity size to 1/3 [4,5].

At optical wavelengths, metamaterials [6,7] also enabled significant improvement in the size reduction of optical cavities [8,9]. Conventional dielectric optical cavities (photonic crystals and micro-disks) have high quality factor Q and diffraction-limited cavity mode volumes (cubic half-wavelength in material), while plasmonic cavities modes can have sub-wavelength volumes (beating the diffraction-limit), but suffer from high losses due to the absorption of metals [10]. Recently, Yang et al. experimentally demonstrated that indefinite metamaterials allow confinement of the electromagnetic field to extremely small space ($\lambda/12$) with

relatively high quality factor [9], exploiting the large wavenumbers accessible because of the hyperbolic dispersion of these materials. However, the fields in such resonators are predominantly located within the metamaterial, which experiences relatively high losses.

Here, we demonstrate the condition required to create sub-wavelength hollow cavities within a circular cross-section metamaterial waveguide. We also numerically show the possibility of creating such a cavity with material parameters that can be achieved using layered media. With proper material choice, this is applicable to the entire electromagnetic spectrum [11]. Compared to plasmonic waveguides, waveguides with metamaterial cladding allow versatility in design, i.e. the propagation constant and decay constant in the cladding can be designed independently. The resonant mode confined in the air-core of these cavities is expected to have lower losses compared to metamaterial-filled cavities.

II. RESULTS AND DISCUSSIONS

It was recently demonstrated that uniaxial metamaterial cladding ($\epsilon_r = \epsilon_\phi \neq \epsilon_z, \mu_r = \mu_\phi \neq \mu_z$) could yield guided modes for core diameters that are extremely subwavelength ($D \ll \lambda/2$) [12-14]. One of the regions identified for extreme sub-wavelength diameters is when $\mu_r \mu_z \rightarrow 1$ ($\mu_r < 0, \mu_z < 0$) or $\epsilon_r \epsilon_z \rightarrow 1$ ($\epsilon_r < 0, \epsilon_z < 0$). This condition is the generalized version of SPP (surface plasmon polariton) resonance in metals [15]. Introducing a discontinuity in the propagation direction or using reflective metallic plates at two ends of the waveguide, shown in Fig. 1, can create a waveguide-based cavity. In this work, we use perfect electric conductor (PEC) for longitudinal confinement and we study the condition (material parameters) required to create a sub-wavelength cavity. A cavity mode is obtained when waves interfere constructively at each round trip in the longitudinal direction:

$$2h = m\lambda_g + 2\varphi_r \quad (1)$$

where h is the resonance length, $\lambda_g = \lambda_0/\bar{\beta}$ is the wavelength in the waveguide, m is an integer number, φ_r is the phase change at reflection (π for PEC), and $\bar{\beta}$ is the normalized propagation constant of the waveguide. As the smallest integer number is unity, $h = \lambda_g/2 = \lambda_0/2\bar{\beta}$ is the shortest achievable resonance length with metallic reflectors. This

indicates we require large normalized propagation constant to reduce the resonance length.

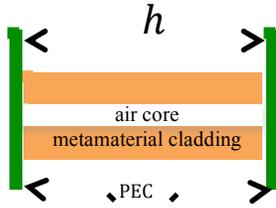


Fig. 1. Schematic of a waveguide-based cavity, where the longitudinal confinement is created using a PEC.

We consider a non-magnetic anisotropic uniaxial cladding material (i.e. $\epsilon_r = \epsilon_\phi \neq \epsilon_z$ and $\mu_r = \mu_z = 1$) with an air-core diameter $D = \lambda/35$ as an example to study the conditions discussed above. Figure 2(a) maps the normalized propagation constant (in logarithmic scale) when the transverse and longitudinal permittivity changes from zero to -2.25. In a waveguide with such a sub-wavelength diameter, no mode exists beyond the white dashed hyperbola, $\epsilon_r \epsilon_z = 1$, as discussed in Refs. [11,12]. The diagonal white dotted line represents points where the cladding material is isotropic, i.e. $\epsilon_r = \epsilon_\phi = \epsilon_z$. The propagation constant of the guided mode diverges when the condition $\epsilon_r \epsilon_z = 1$ is reached. We observe that $\bar{\beta}$ diverges faster along the horizontal axis ($\epsilon_r = -2.25$) compared to $\bar{\beta}$ along the vertical axis ($\epsilon_z = -2.25$) and isotropic case (the dotted line). This indicates that at the vicinity of the limiting hyperbola the resonant length will approach zero.

A layered medium consisting of layers of gold and dielectric ($n=1.5$) [11], with a 0.3 dielectric volume fraction, represents a uniaxial metamaterial with material properties $\epsilon_r \approx -0.38, \epsilon_z \approx -2.25$ at 300 nm wavelength. We use CST Microwave Studio 2015 to investigate a sub-wavelength cavity numerically with such material parameters. The cavity is consisting of an air hole with a finite metamaterial cladding (10 times greater than the core diameter). We used PEC to create the longitudinal confinement. We observe a good agreement (<1% difference) between analytical and numerical results for a cavity with $D/\lambda > 0.15$ and $h/\lambda = 0.03$ leading to $D^2 h/\lambda^3 = 6.7e-4$. Figure 3 (b) and (c) show the electric and magnetic fields of the cavity when $D/\lambda = 0.17$ and $h/\lambda = 0.03$. The black lines represent the air core area. The two vertical rectangles are air regions separated so that finer mesh can be achieved close to the core-cladding interface where the fields decay very fast.

In conclusion, we have shown that using anisotropic metamaterial one can create air-core sub-wavelength cavities when $\mu_r \mu_z \rightarrow 1$ ($\mu_r < 0, \mu_z < 0$) or $\epsilon_r \epsilon_z \rightarrow 1$ ($\epsilon_r < 0, \epsilon_z < 0$). We demonstrate that sandwiching an air hole within a layered medium between two PECs can lead into a cavity with $D^2 h/\lambda^3 = 6.7e-4$.

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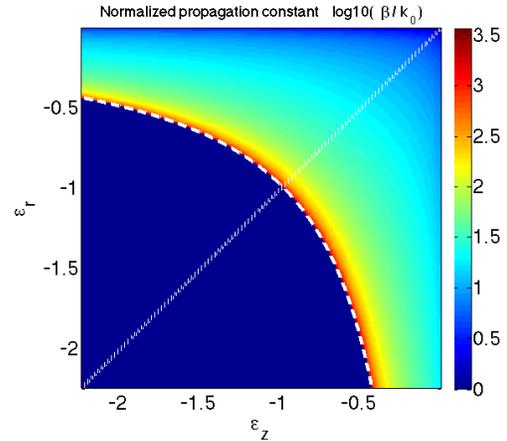


Fig. 2. Normalized propagation constant as a function of ϵ_r and ϵ_z of a hollow-core waveguide with uniaxial metamaterial cladding.

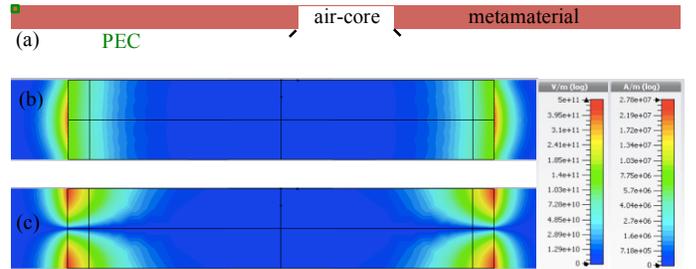


Fig. 3. (a) Electric and (b) magnetic field magnitude in a sub-wavelength cavity. The metamaterial cladding is 10 times of the core diameter.

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