# Exact numerical modelling for finite samples of discrete metamaterials

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***Abstract*—I will present the details of an exact numerical approach for precise modelling of artificial magnetic metamaterials, applicable for microwave and radio-frequency range. The pre-requisite for this modelling is a structure assembled as an array of capacitively-loaded well-conducting rings, which is most typical for microwave applications of metamaterials. The exact calculation takes all the mutual interactions into account, however a number of time-saving symmetry considerations can be applied to calculate the total impedance matrix.**

## I. INTRODUCTION

Metamaterials are usually described in terms of effective material parameters [1]–[15], however it is known [16] that the real performance of practical metamaterial devices significantly deviates from theoretical predictions, even for strongly subwavelength systems. One of the reasons for that discrepancy is the finite size and finite number of individual structural elements (unit cells of metamaterial). To analyse the response of finite metamaterials with discrete structure reliably, and yet to avoid the approach of full-wave numerical simulations, a semi-analytical theory was developed, based on the circuit modelling of the structure.

I will present the details of this approach, as applied to artificial magnetic metamaterials, based on capacitively-loaded conducting rings [17]. The exact calculation takes all the mutual interactions into account, however a number of time-saving symmetry considerations can be applied to calculate the total impedance matrix.

I will then report the outcomes of such modelling with regards to the realistic metamaterial structures, and demonstrate some important differences as compared to the design predictions. More specifically, I will report new findings related to the effect of a discrete structure of practical metamaterials, as opposed to the homogenised treatment assumed in the effective medium treatment.

Indeed, one of the newly found aspects [18] is that boundary effects play a dramatic role in finite metamaterial samples with discrete structure, making their observable properties quite different from the predictions of effective medium theory. In particular, general effective medium treatments, even those tailored for a finite-thickness slabs [19], failed to describe the observable properties of metamaterial lenses limited in all the three dimensions [20].

## II. RESULTS

We now analyse the convergence of the actual properties of discrete structures towards a homogenised response, taking a spherical shape of metamaterial sample (a cubic lattice, truncated to a shape as close to a sphere as possible). For small spheres with just a few unit cells along the diameter the shape

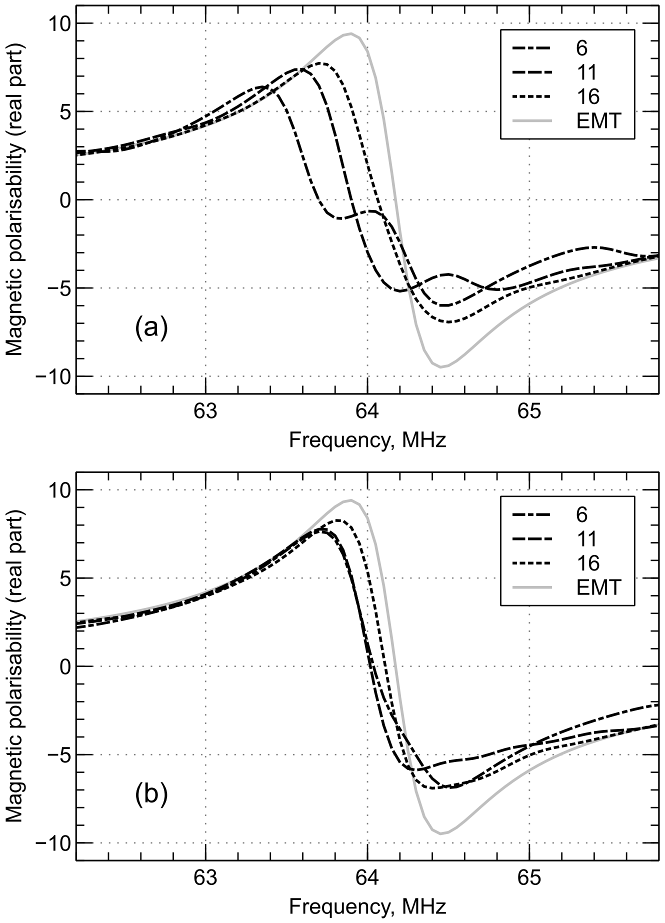

Fig. 1. Frequency dependence of the real part of the magnetic polarisability of the quasi-spherical metamaterial samples truncated from (a) “flat” or (b) “ragged” configuration of the initial boundary of the cubes. The sizes of the spheres, in terms of unit cells per diameter, is indicated by the numbers in the insets. The grey solid curve shows the polarisation theoretically calculated for a homogeneous sphere with the effective permeability [21] corresponding to the considered metamaterial structure.

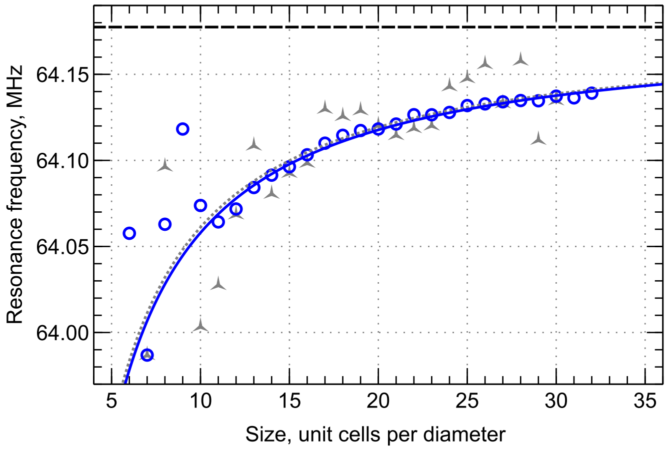

Fig. 2. Resonance frequency of the magnetic polarisability of the discrete spherical samples, depending on their size (symbols), and the corresponding convergence fits (lines), for the case of uniaxial structure with regular (blue circles, solid line) or low (grey stars, dotted line) dissipation. Theoretical frequency of the resonance is shown by black horizontal dash.

is remarkably ragged, however larger spheres appear reasonably smooth overall, with a good visual spherical appearance for sizes exceeding about 15 unit cells per diameter.

We directly calculate the response of this structure to applied field, taking all the mutual interactions between the loops into account [22]. We have observed that the calculated magnetisation curves (Fig. 1) for small discrete samples show remarkable deviations and less trivial frequency dependence, however the convergence towards the continuous model improves with size and becomes a clear trend for sizes above 11, and the results for the spheres of 16 and larger appear very similar to each other.

Although we have no computational tools to calculate much larger samples, the analysis of the convergence trend towards the effective medium theory (Fig. 2) allows us to conclude that eventually the difference between a discrete sphere and a continuous one can be eliminated to good precision.

## III. CONCLUSION

The effects outlined above are particularly prominent in metamaterials with strongly interacting elements, such as those based on ring resonators. It also appears that having a resonance is essential for boundary effects to spread through the structure. Indeed, as opposed to resonant metamaterials, response of artificial diamagnetics of finite size and discrete structure is in a good agreement with the effective medium predictions. Our conclusions are likely to be rather general, applicable in a wide frequency range and for many specific designs, so these results may have severe implications for practical development of metamaterials.

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