Spatio-Temporal Pulse Propagation in nonlinear dispersive optical Media

Carsten Brée, Shalva Amiranashvili and Uwe Bandelow
Weierstrass Institute for Applied Analysis and Stochastics, Mohrenstr. 39, 10117 Berlin, Germany
Email: see http://www.wias-berlin.de/contact/staff

***Abstract*—We discuss models for the propagation of ultrashort optical pulses through nonlinear dispersive optical media. Starting from a single-mode fiber with fixed radial field structure and one propagation coordinate we turn to a full three-dimensional model for propagation of ultrashort pulses in gases.**

I. Introduction

Ultrashort laser pulses have dramatically triggered both fundamental and applied science and also created new challenges from the numerical side. A straightforward solution of the underlying field and material equations becomes impractical because too different space- and time-scales are involved, and the common slowly varying envelope approximation (SVEA) is no longer valid for ultrashort pulses. Therefore new models which allow for an efficient numerical treatment have to be developed [1], [2]. We discuss several such models starting from the case of a single-mode waveguide in which the field structure in the radial direction is fixed and only one propagation coordinate is involved [3]–[7]. Thereafter we turn to the full three-dimensional modeling of propagation of ultrashort pulses in gases [8].

II. Scalar Case

We start with an exemplary straightforward numerical solution for an ultrashort pulse propagating in a single-mode fiber [4], Fig. 1. As shown there, the envelope structure is destroyed in the course of propagation. Another observation is that the pulse carrier frequency is shifted and therefore not well defined. Such extreme propagation regimes require more careful treatment than the traditional envelope description.

In principle, an optical pulse in a single-mode waveguide can be described by a single field component $E(\vec{r},t)$, which, to a good approximation, is governed by a scalar wave equation

$$(\partial_z^2+\vec{\nabla}_\perp^2)E-\frac{1}{c^2}\partial_t^2\left(\hat{\epsilon}E\right)=\mu_0\partial_t^2P_{\mathrm{NL}},\quad \vec{\nabla}_\perp^2=\partial_x^2+\partial_y^2, \quad (1)$$

where the dispersion operator $\hat{\epsilon}$ is defined in the frequency domain $\left(\hat{\epsilon}E\right)_\omega=\epsilon(\omega)E_\omega$ and P_{NL} denotes the nonlinear part of the induced polarization. We decompose the real-valued electric field $E(\vec{r},t)=\sum_\omega E_\omega(\vec{r})e^{-i\omega t}$, $E_{-\omega}=E_\omega^*$, into the complex-valued negative- and positive-frequency parts

$$E=\sum_{\omega<0}E_\omega e^{-i\omega t}+\sum_{\omega>0}E_\omega e^{-i\omega t}=E^{(-)}+E^{(+)},$$

introduce the analytic signal $\mathcal{E}$ for the electric field $E=\mathrm{Re}[\mathcal{E}]$

$$\mathcal{E}(\vec{r},t)=2E^{(+)}(\vec{r},t)=2\sum_{\omega>0}E_\omega(\vec{r})e^{-i\omega t},$$

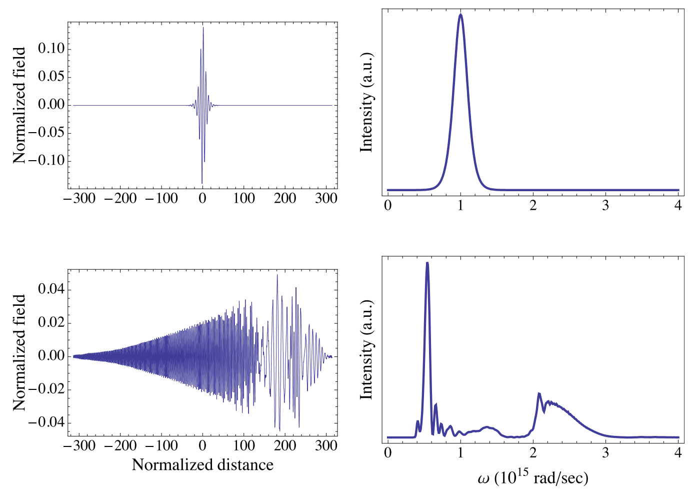

Fig. 1. Top: electric field (left) and spectrum (right) of the initial pulse. Bottom: the same after 10 ps propagation in a bulk fluoride glass. One sees that the envelope structure of the initial pulse is gradually destroyed in the course of propagation.

and obtain in the unidirectional approximation

$$\left(i\partial_z+\hat{\beta}\right)\mathcal{E}+\frac{\hat{\beta}^{-1}}{2}\vec{\nabla}_\perp^2\mathcal{E}=\frac{3\chi^{(3)}}{8c^2}\hat{\beta}^{-1}\partial_t^2\left(|\mathcal{E}|^2\mathcal{E}\right)^{(+)}. \quad (2)$$

Equation (2) is similar to the nonlinear Schrödinger equation (NSE). However, Eq. (2) is completely independent on SVEA. If the pulse can be characterized by a narrow spectrum around the carrier frequency ω_0 and the corresponding wave vector $\beta_0=\beta(\omega_0)$, we get the equation

$$\left(i\partial_z+\hat{\beta}\right)\mathcal{E}+\frac{1}{2\beta_0}\vec{\nabla}_\perp^2\mathcal{E}+\frac{3\omega_0\chi^{(3)}}{8cn(\omega_0)}|\mathcal{E}|^2\mathcal{E}=0,$$

which can be related to the 1D NSE for the pulse envelope ψ by a standard transformation to the pulse-comoving frame

$$\mathcal{E}(\vec{r},t)=\Re(x,y)\psi(z,\tau)e^{i(\beta_0z-\omega_0t)},\qquad \tau=t-\beta_1z,$$

where $\beta_1=\beta'(\omega_0)$ is the reverse group velocity and $\Re(x,y)$ is the transverse mode profile [9]. A similar elimination of the radial coordinates can be appled directly to Eq. (2).

III. Vectorial case

In a homogeneous medium without (linear) waveguiding one has to account, in principle, for a fully vectorial description of the electric field $\vec{E}$, e.g., in the frequency domain by

$$[\vec{\nabla}^2+\beta^2(\omega)]\vec{E}_\omega=\vec{S}_\omega. \quad (3)$$

978-1-4673-1604-0/12/$31.00 ©2012 IEEE

The source term $\vec{S}_\omega$ is given by

$$\vec{S}_\omega = -\mu_0\omega^2\vec{P}_{\mathrm{NL},\omega} + i\mu_0\omega\vec{J}_\omega + \frac{1}{\epsilon_0}\vec{\nabla}\left(\rho - \vec{\nabla}\cdot\vec{P}_\omega\right), \quad (4)$$

and takes account of the nonlinear part $\vec{P}_{\mathrm{NL},\omega}$ of the total polarization density $\vec{P}_\omega$, the existence of free carriers with density ρ and current density $\vec{J}_\omega$, respectively. The last term on the r.h.s. of Eq. (4) models vectorial effects which become important for strongly divergent beams occuring under extreme focusing conditions. Scalar approximations can be restored for many experimental situations of interest. Nonlinear self-focusing effects may increase the optical intensity to trigger photoionization, which requires to include free carrier terms. We obtain a set of coupled equations for forward and backward electric field components

$$(i\partial_z \pm |k_z|)\vec{\mathcal{E}}^{\pm}_\omega = -\mu_0\omega^2\left[\mathbf{1} - \frac{\vec{k}\otimes\vec{k}}{k^2}\right](\vec{P}_{\mathrm{NL},\omega} + i\vec{J}_\omega/\omega). \quad (5)$$

As the operator $\mathbf{1} - \vec{k}\otimes\vec{k}/k^2$ projects out longitudinal field components, the evolution of the latter is governed by a source-free equation, while transverse components are governed by Eq. (5). Due to the presence of $\mathbf{1} - \vec{k}\otimes\vec{k}/k^2$, this bidirectional equation requires very costly numerics. The scalar, unidirectional limit $\vec{\mathcal{E}} \to \mathcal{E}$ comparable to Eq. (2) is obtained by letting

$$\frac{\vec{k}\otimes\vec{k}}{k^2}(\vec{P}_{\mathrm{NL},\omega} + i\vec{J}_\omega/\omega) \approx 0, \qquad \vec{\mathcal{E}}^{-} \approx 0, \quad (6)$$

i.e. by neglecting longitudinal field components and decoupling orthogonal polarization states, as well as by dropping backward propagating waves. However, while in the case Eq. (2) forward propagating field components can be identified with the positive frequency part of the electric field, this correspondence breaks down in the non-waveguiding case. It may be restored under the paraxial approximation $k_\perp \ll k_z$. In the unidirectional limit, this definition of directional fields leads to the forward Maxwell equation (FME) [10], which is successfully used in the context of femtosecond filamentation.

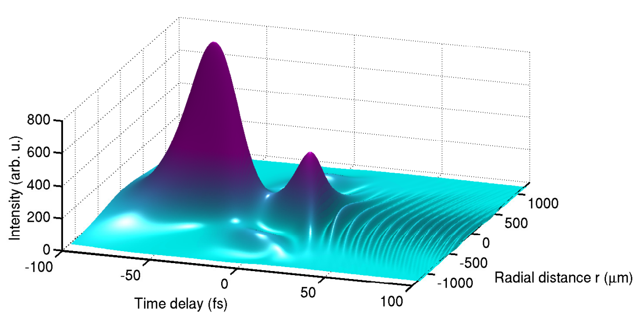

Fig. 2. Numerical simulation of optical wavebreaking in a femtosecond filament in argon.

The formation of femtosecond filaments can be observed when pulse femtosecond laser radiation is loosely focused into a dielectric medium, provided that the peak optical power P of the pulses exceeds a certain critical threshold P_{thr}. These filaments are narrow, longitudinally extended structures of dilute plasma and light. While for moderate ratios P/P_{thr} the cylindrical symmetry of the input beam can be maintained, an azimuthal modulation instability leads to multifilamentation and loss of cylindrical symmetry for higher input powers. The conservation of cylindrical symmetry leads to a strong simplification of the required numerics and holds for many interesting scenarios like pulse self-compression and harmonic generation within filaments. Furthermore, in these cases it is often justified to work within the paraxial approximation, such that the evolution of the electric field is governed by the FME. Numerically, this is solved using a pseudospectral split-step scheme, where that part of the propagation equation governed by the radial component $\Delta_r = 1/r\partial_r r\partial_r$ of the Laplacian is discretized using an implicit Crank-Nicolson scheme in the frequency domain, while the nonlinear part of the FME is evaluated in the time domain, involving the need for repeated Fourier transforms between temporal and spectral domain and accounting for aliasing errors. A characteristic radially symmetric field arising in a simulation of femtosecond filamentation is shown in Fig. (2) [11], which depicts the higher dimensional analogue of optical wavebreaking occuring during fiber propagation [9]. It is caused by modulation instability and is also known as hyperbolic shock-wave formation [12].

Acknowledgment

The work of Sh. Amiranashvili has been supported by the Deutsche Forschungsgemeinschaft (DFG) within the collaborative research center MATHEON under Grant D14.

References

[1] T. Brabec and F. Krausz, "Nonlinear optical pulse propagation in the single-cycle regime," *Phys. Rev. Lett.*, vol. 78, no. 17, p. 3282, 1997.
[2] M. Kolesik and J. V. Moloney, "Nonlinear optical pulse propagation simulation: From Maxwell's to unidirectional equations," *Phys. Rev. E*, vol. 70, no. 3, p. 036604, 2004.
[3] S. Amiranashvili, U. Bandelow, and A. Mielke, "Pade approximant for refractive index and nonlocal envelope equations," *Opt. Commun.*, vol. 283, pp. 480–485, 2010.
[4] S. Amiranashvili, A. G. Vladimirov, and U. Bandelow, "A model equation for ultrashort optical pulses around the zero dispersion frequency," *Europ. Phys. J. D*, vol. 58, pp. 219–226, 2010.
[5] S. Amiranashvili and A. Demircan, "Hamiltonian structure of propagation equations for ultrashort optical pulses," *Phys. Rev. A*, vol. 82, no. 1, p. 013812, 2010.
[6] S. Amiranashvili, U. Bandelow, and A. Mielke, "Calculation of ultrashort pulse propagation based on rational approximations for medium dispersion," *Opt. Quantum Electron.*, vol. Online First, 2012.
[7] S. Amiranashvili and A. Demircan, "Ultrashort optical pulse propagation in terms of analytic signal," *Advances in Optical Technologies*, vol. 2011, p. 989515, 2011.
[8] L. Berge, S. Skupin, R. Nuter, J. Kasparian, and J. P. Wolf, "Ultrashort filaments of light in weakly ionized, optically transparent media," *Rep. Prog. Phys.*, vol. 70, p. 1633, 2007.
[9] G. P. Agrawal, *Nonlinear Fiber Optics*, 3rd ed., ser. Optics and Photonics. Academic Press, 2001.
[10] A. V. Husakou and J. Herrmann, "Supercontinuum generation of higher-order solitons by fission in photonic crystal fibers," *Phys. Rev. Lett.*, vol. 87, no. 20, p. 203901, Oct 2001.
[11] C. Brée, J. Bethge, S. Skupin, L. Bergé, A. Demircan, and G. Steinmeyer, "Cascaded self-compression of femtosecond pulses in filaments," *New J. Phys.*, vol. 12, p. 093046, 2010.
[12] L. Berge, K. Germaschewski, R. Grauer, and J. J. Rasmussen, "Hyperbolic shockwaves of the optical self-focusing with normal group-velocity dispersion," *Phys. Rev. Lett.*, vol. 89, p. 153902, 2002.