Optical design of a qubit embedded in photonic crystals for rotation gate operations

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Abstract- We have optimized the optical design parameters of a qubit composed of excited states of an atom embedded in photonic crystals for operating rotation gates, using the coherent control of spontaneous emission from the atom.

I. INTRODUCTION

Very small defects (such as quantum dots or impurity atoms) embedded in photonic crystals can provide a promising building block of optical solid-state devices for quantum information processing [1]. Near the embedded atom, light is confined due to the presence of a photonic bandgap (PBG), which leads to the formation of a photon-atom bound state with nonzero steady-state atomic populations on excited states [2]. This may provide the basis of a qubit encoding optical and quantum information. Furthermore, robust mechanisms for processing optical and quantum information have been proposed, such as the population switching of an atom (or a quantum dot) embedded in photonic crystals [3]. So far, we have also demonstrated the coherent control of spontaneous emission from such an embedded atom with nonzero steady-state atomic populations [4]. Using the coherent control, we have also clarified a method of confining light in a single mode near the embedded atom, which is useful for a qubit [5]. A recent major theoretical challenge in this field is the investigation of carrying out a quantum logic gate operation to a qubit, especially a single-qubit rotation gate, which is one of the universal quantum logic gates and a key operation for quantum computing.

In this paper, to carry out near-complete rotation gate operation, we clarify an optical design of a solid-state qubit composed of excited states of an impurity atom embedded in photonic crystals, using the coherent control of spontaneous emission from the embedded atom, where we optimize the optical design parameters such as control laser strength and the transition frequency of the embedded atom.

II. QUANTUM DOT IN PHOTONIC CRYSTALS

We let $|0\rangle$ and $|1\rangle$ denote two excited states of an impurity three-level atom embedded in photonic crystals with a high-pass 3D PBG. It is assumed that the transition frequency between $|1\rangle$ and the ground state $|g\rangle$ (ω_{1g}) is far inside the PBG and that the transition frequency between $|0\rangle$ and $|g\rangle$ (ω_{0g}) is near the band edge frequency ω_C, where the detuning is denoted as $\delta = \omega_{0g} - \omega_C$. The coherent superposition of the far-inside bit $|1\rangle$ and the edge-side bit $|0\rangle$

$$|\psi(t)\rangle = a_0(t)|0\rangle + a_1(t)|1\rangle \qquad (1)$$

can act as a qubit with a complex amplitude $a_k(t)$ (k=0, 1), which provides the atomic population $n_k(t)=|a_k(t)|^2$ (namely, the probability of finding the atom in level $|k\rangle$). The time evaluation of the qubit can be shown by the Bloch sphere representation, as shown in Fig. 1. Here, the position of the qubit is determined by the norm $N(t)=n_0(t)+n_1(t)$ (the length from the origin that is given by the sum of the atomic populations on excited states), the ratio $\tan\theta(t)=|a_0(t)|/|a_1(t)|$, and the phase difference $\phi(t)=Arg(a_0(t)/a_1(t))$.

At the initial time (t=0), using an ultrashort pumping laser pulse, the atom is excited ($N(0)=1$), where the initial phases $\theta(0)$ and $\phi(0)$ are determined by the pulse area and phase of the laser pulses. For $t>0$, a coherent laser is used for coupling the bits $|0\rangle$ and $|1\rangle$, where the strength of the control laser is characterized by the Rabi frequency Ω. Here, the complete rotation gate operation requires qubit evaluation on the circle C in Fig. 1(a), which is determined by $N(t)=1$ and $\tan\theta(t)=1$.

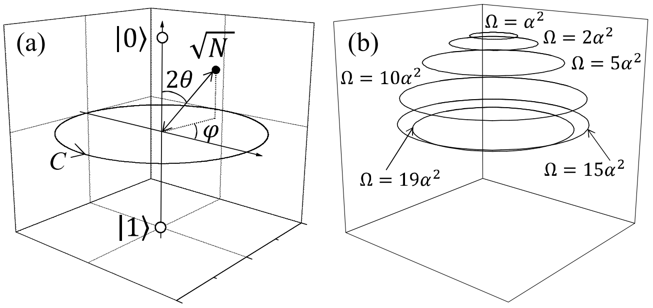

Fig. 1. Bloch sphere representation:
(a) the position of a qubit and (b) the qubit evaluation paths.

Fig. 1(b) shows the qubit evaluation paths in the Bloch sphere for various Rabi frequencies Ω and $\delta = -20\alpha^2$ with a scaled parameter α [3], where we apply the condition for making confined light composed of a single localized mode, which is fulfilled by preparing the initial atomic state to satisfy the conditions for the initial phases $\theta(0)$ and $\phi(0)$ that have been reported in Ref. [5]. Under these conditions, in addition to the nonzero steady-state norm, the population ratio $\tan\theta(t)$ remains constant, leading to the circular rotation, due to the lack of dynamical transfer of populations between the two bits $|0\rangle$ and $|1\rangle$, because Rabi splitting disappears with the destructive quantum interference. Furthermore, we find that the qubit evaluation paths approach the circle C with increasing Ω. However, to further increase Rabi frequency ($\Omega=19\alpha^2$),

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the norm is reduced, leading to the loss of the quantum information. So, we note that there are optimum values of the optical design parameters (the Rabi frequency Ω and the detuning δ characterizing control laser strength and the transition frequency of the atom, respectively) for rotation gate operations ($N(t)$=1 and $\tan\theta(t)$=1).

III. OPTIMIZATION OF OPTICAL DESIGN PARAMETERS

First, we consider the dependence of the norm on the optical design parameters. In Fig. 2, we plot the steady-state norm N_S=lim $N(t)$ ($t \to \infty$) as a function of the Rabi frequency for various values of the detuning δ. Fig. 2 shows that the norm N_S is increased by increasing the detuning $|\delta|$, namely, by moving the transition frequency ω_{0g} far inside the PBG, accompanied by the enhancement of the light confinement effect. On the other hand, by increasing the large Rabi frequency, the norm N_S is decreased, and then the norm N_S decays to zero at the cutoff $\Omega_C(\delta)$, which is given by $\Omega_C(\delta) \cong |\delta|$. In the region $\Omega_C(\delta) < \Omega$, due to the Stark shift caused by the control laser, the edge-side bit $|0\rangle$ (ω_{0g}) moves out of the PBG, and then in addition to $n_0(t)$, the atomic population of the far-inside bit $|1\rangle$ ($n_1(t)$) completely decays to the ground state $|g\rangle$ through the $|0\rangle - |1\rangle$ channel coupled by the control laser. Therefore, we note that the condition $\Omega < \Omega_C(\delta)$ is required to maintain the quantum information.

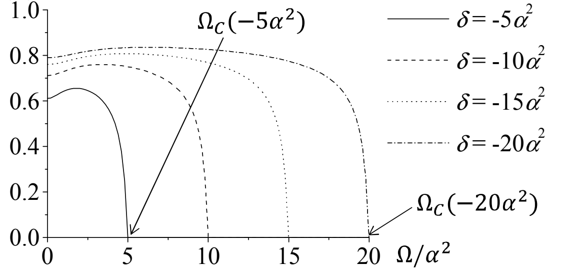

Fig. 2. Steady-state norm as a function of the Rabi frequency.

In Fig. 3, we plot the steady-state population ratio $\tan\theta_S$ =lim $\tan\theta(t)$ ($t \to \infty$). The steady-state population of the bit $|1\rangle$ (n_{1S}) is nearly constant with varying the detuning δ, because the bit $|1\rangle$ is far inside the PBG, whereas that of the edge-side bit $|0\rangle$ (n_{0S}) increases with increasing the detuning $|\delta|$, which leads to a decrease in the ratio $\tan\theta_S$ (but $\tan\theta_S$ >0 is maintained). On the other hand, the ratio $\tan\theta_S$ increases with increasing the Rabi frequency, and then the ratio approaches 1. This is because the increase in the Rabi frequency facilitates the population transfer between the bits $|0\rangle$ and $|1\rangle$, which equalizes their populations. At $\Omega = \Omega_C(\delta)$, the ratio is 1. Therefore, we note that the ratio satisfies 0< $\tan\theta_S$ <1 in the region where the quantum information is maintained ($\Omega < \Omega_C(\delta)$).

Consequently, we find that both the norm and the ratio satisfy 0<(N_S and $\tan\theta_S$)<1, so that the optimization of the optical design parameters (the Rabi frequency and the detuning) for nearly complete rotation gate operations ($N_S \cong 1$ and $\tan\theta_S \cong 1$) is given by a pair of Ω and δ that maximizes $P=N_S+\tan\theta_S$. Fig. 4 shows a contour plot of P for $\Omega < \Omega_C(\delta)$ and $-20\alpha^2 < \delta < 0$. This figure indicates that P has a ridge fitting to $\Omega_F(\delta)$ with R^2=0.99 from our numerical calculations. This line $\Omega_F(\delta)$ provides the optimum Rabi frequencies (control laser strengths). As a result, we find that, for the nearly complete rotation gate operations, the transition frequency lying far inside the PBG is an appreciable choice in suppressing the loss of quantum information, while the control laser strength should be managed as $\Omega_F(\delta)$ to enhance the population transfer in the two bits for $\tan\theta_S \cong 1$ and to suppress the Stark shift for $N_S \cong 1$.

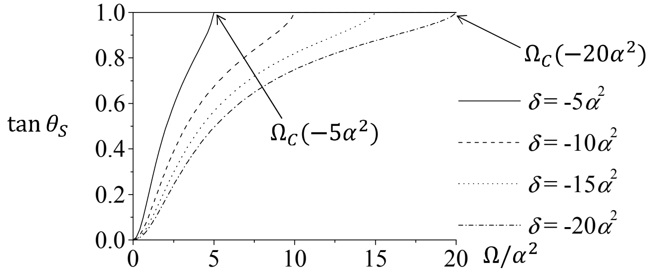

Fig. 3. Steady-state population ratio as a function of the Rabi frequency.

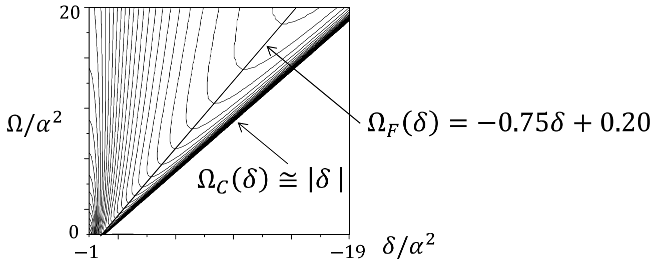

Fig. 4. Contour plot of P.

IV. CONCLUSION

We have optimized the atomic transition frequency and control laser strength for operating nearly complete rotation gates for a qubit composed of excited states of an atom embedded in photonic crystals, using the coherent control of spontaneous emission from the embedded atom. The result of this study is useful for determining the designs of future quantum logic gates based on solid-state photonic crystal systems.

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