

# Nonlinear optics in photonic crystal nanostructures

(Invited Paper)

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**Abstract**—We examine nonlinear Kerr effects in slow-light photonic crystals in the presence of multi-photon absorption and free-carriers. We derive analytic formulations for self-phase modulation limited by three-photon absorption. These observations are confirmed with a modified nonlinear Schrödinger equation (NLSE). Experimental verifications of several nonlinear processes are presented to support the modeling.

## I. INTRODUCTION

Slow-light enhanced nonlinearities have been studied extensively in semiconductor photonic crystals as key elements to future photonic technologies [1]. A variety of nonlinear processes such as temporal soliton compression [2], four-wave mixing [3], Raman scattering [4], third-harmonic generation [5], and self-phase modulation (SPM) [6], [7] have been demonstrated in photonic crystal waveguides (PhCWG). A key challenge in these works is multi-photon absorption mechanisms such as two-photon absorption (TPA) and three-photon absorption (ThPA) which restrict the desirable Kerr effect. Additionally, TPA and ThPA generate free carriers that induce both free-carrier absorption (FCA) and free-carrier dispersion (FCD). This latter effect is particularly detrimental to the propagating pulse shape. Here we present an analytic formulation of slow-light SPM for materials limited by ThPA as a leading term and compare it to TPA-restricted materials [8]. We derive critical intensity thresholds,  $I_c$ , at which FCD degrades the pulse propagation. Though the analysis here focuses on the 1.55  $\mu\text{m}$  wavelength range, the results in this work are applicable to any material waveguide system limited by three-photon absorption. In particular, several groups have recently initiated systematic investigation of nonlinear optics in silicon near 2  $\mu\text{m}$  where TPA is drastically reduced [9], [10].

## II. NONLINEAR WAVE PROPAGATION IN SLOW-LIGHT SEMICONDUCTORS

The propagation of picosecond optical pulses in a waveguide with suppressed TPA and negligible group velocity dispersion is governed by [7], [11], [12]:

$$\frac{\partial E}{\partial z} = -\frac{\alpha}{2}E + ik_0 n_2 |E|^2 E - \frac{\alpha_3}{2} |E|^4 E + (ik_o \frac{dn}{dN} - \frac{\sigma}{2}) N_c E \quad (1)$$

where  $E$  is the electric field envelope (Intensity  $I_o = |E|^2$ ),  $\alpha$  the linear loss,  $k_0 = 2\pi/\lambda$ ,  $n_2$  the optical Kerr coefficient,  $\alpha_3$  the ThPA coefficient,  $dn/dN$  the index change per carrier density,  $\sigma$  the FCA coefficient,  $N_c$  the number of carriers, and  $z$  the distance along the waveguide of length,  $L$ . The

carrier equation:  $\frac{\partial N_c(z,t)}{\partial t} = \frac{\alpha_3}{3\hbar\omega} |E(z,t)|^6 - \frac{N_c(z,t)}{\tau_c}$ , describes free-carriers generated by ThPA, as well as recombination with lifetime,  $\tau_c$ . We also extend this formalism to include slow group velocity enhancement of the optical field. Though slow-light effects physically affect the field intensity, here we attach the scalings to the coefficients for notational simplicity: [Kerr]  $n_{2eff} = n_2(n_g/n_0)^2$ ; [TPA]  $\alpha_{2eff} = \alpha_2(n_g/n_0)^2$  [1]; [ThPA]  $\alpha_{3eff} = \alpha_3(n_g/n_0)^3(1/A_{5eff})^2$  [7], with  $A_{5eff}$  the 5<sup>th</sup>-order area,  $\alpha_2$  the two-photon absorption coefficient, and  $n_g$  the slow-light group index.

In Fig. 1(a) we plot phase shift,  $\phi$ , as a function of  $I_0$  for representative materials, GaInP (ThPA) [2], [7], and silicon (TPA) [6], with  $n_g = 20$ ,  $L = 1$  mm, and  $\alpha = 10$  dB/cm, achievable in PhCWGs at present. The solid lines indicate the analytic formulation of  $\phi$  according to the pertinent formulas in [8], [11], while the straight (dashed) lines serve as a reference in the absence of nonlinear loss. The ThPA-limited material GaInP demonstrates larger phase shift compared to TPA-limited Si. We also comment on the energy implications of these slow-light structures [13].

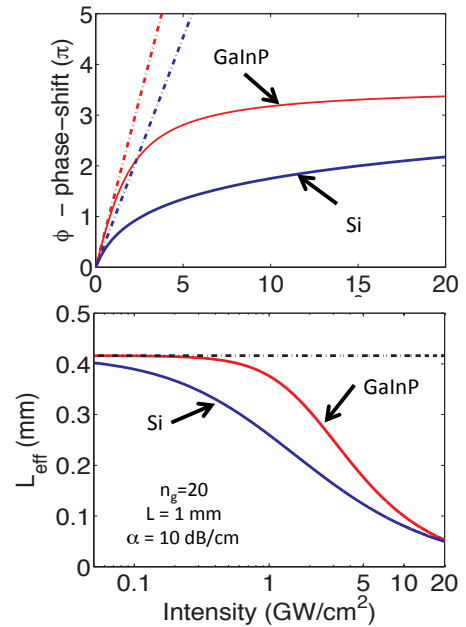


Fig. 1. (a) Phase shift,  $\phi$ , as a function of  $I_o$  for (i) GaInP and (ii) Si. The straight dashed lines are  $\alpha$  only, while the curves shows  $\phi$  impacted by TPA or ThPA. (b) Effective lengths,  $L_{eff}(TPA)[\text{Si}]$  and  $L_{eff}(ThPA)[\text{GaInP}]$  vs.  $I_o$  with  $L_{eff}(lin.)$  shown (dashed black line) as reference.

In conventional optical fibers with small nonlinearities,  $L_{eff}(lin.) = (1 - \exp[-\alpha L])/\alpha$ , e.g. no intensity dependence. In the case of materials with nonlinear absorption, different definitions of effective length should be used:  $L_{eff}(TPA)$ [11] or ThPA,  $L_{eff}(ThPA)$ , readily obtainable from re-arranging the phase equation in [8]. In Fig. 1(b), we show the corresponding  $L_{eff}$  versus  $I_0$ . While  $L_{eff}(TPA)$  deviates almost immediately, note that  $L_{eff}(ThPA) \approx L_{eff}(lin.)$  up to about  $I_0 = 0.3 \text{ GW/cm}^2$  for  $n_g=20$ , and is still 90% of  $L_{eff}(lin.)$  at  $1 \text{ GW/cm}^2$ , indicating that nonlinear losses are weak under these conditions. This is in sharp contrast to the TPA material, which falls off immediately. While multi-photon absorption restricts SPM, and other nonlinear effects in general, the far greater impediment is free-carrier effects.

We plot the temporal pulse properties in Fig. 2(a). While  $\phi = 1.5\pi$  is observed with only a slight onset of blue-shift for GaInP, the pulse undergoes a dramatic blue-shift in Si due to a greater number of free-carriers generated at this power level. The spectral properties in Fig. 2(b) show slight asymmetry in GaInP, as expected above a critical intensity  $I_c$  [8]. A much smaller phase shift with a strong trailing blue component are apparent in the case of Si.

We describe modeling and experimental verification of nonlinear soliton compression in PhCWGs with a modified nonlinear Schrödinger equation (NLSE) [2]. Fig. 3 shows the modeled and experimental pulse temporal compression for an example data set. Importantly, the model leads to simultaneous agreement with both the autocorrelation and spectra with no degrees of freedom. The insets of Fig. 3 show the compression factor  $\chi_c = T_0/T_{comp}$  and the compression quality factor  $Q_c = P_{peak}/\chi_c$ , where  $P_{peak}$  is the output peak intensity normalized to the input peak intensity.

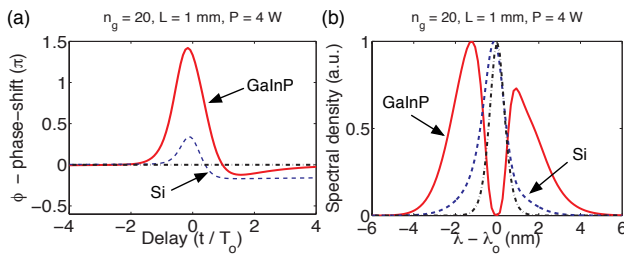


Fig. 2. (a) Phase shift ( $\pi$ ) vs. delay for  $P = 4 \text{ W}$  at  $n_g=20$  for GaInP, solid, and Si dashed (b) Spectra corresponding to panel (a).

### III. CONCLUSION

We presented analytic and numerical analysis of nonlinear Kerr effects in slow-light photonic crystals with multi-photon absorption and free-carriers. The results are modelled with a modified nonlinear Schrödinger equation (NLSE) and confirmed experimentally.

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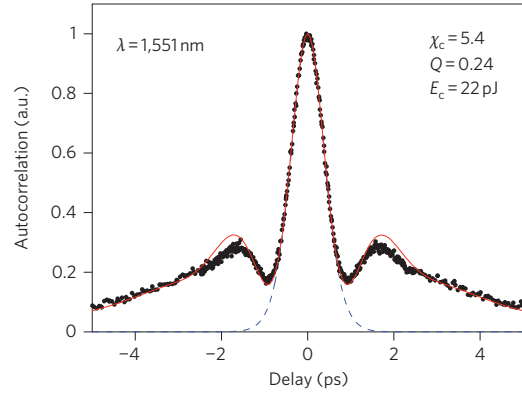


Fig. 3. Calculated (red solid line) and measured autocorrelation traces (black dots) autocorrelation traces at 1551 nm with a coupled pulse energy of 22 pJ together with a hyperbolic secant fit (blue dashed line).

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