

# General design of compact waveguide coupler with homogeneous media

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## Abstract

**Based on finite embedded coordinate transformation method, a general transformation is given to design a compact waveguide coupler. It removes the limitation of material inhomogeneity and makes the coupler be more easily realized. It also offers us much freedom and flexibility in choosing the appropriate material parameters to practically implement the device. Full wave simulation based on the finite element method is performed to confirm the performance of the waveguide coupler.**

## I. INTRODUCTION

Recently, transformation optics has attracted tremendous attention in the world since it provides scientists and engineers with previously unavailable ability and flexibility to manipulate the propagating behavior of the electromagnetic (EM) wave and control the spatial distribution of EM fields. It fully takes the advantage of the form invariance of Maxwell equations under spatial coordinate transformations with the material parameters in the physical space being anisotropic and inhomogeneous. Transformation optics technique is firstly adopted to design an invisibility cloak [1-5], which can guide incident wave smoothly streaming around the object so that observer outside the cloak cannot detect it. The rapid development of metamaterial makes the practical realization of an invisibility cloak become possible. The first concept cloak with reduced material parameters was practically implemented with ten concentric split-ring resonators at microwave frequency [3]. Besides the invisibility cloak, it is also employed to design many novel devices, such as EM field concentrator [6], EM field rotator [7], and illusion optics device [8].

More recently, finite-embedded coordinate transformation has been introduced by Rahm and utilized in the design of beam shifter and splitter [9]. It also led to some other interesting device, such as waveguide expander and compressor [10], waveguide bender [10, 11], and waveguide coupler [12-14]. However, waveguide couplers shown in Refs [12-14] require anisotropic and inhomogeneous medium, which bring obstacles in practical realization of the device. To circumvent the limitation of the material inhomogeneity, Xu et al proposed a new transformation inspired by the design of the carpet cloak introduced by Chen et al [15, 16]. In this work, a general transformation is introduced to achieve the compact waveguide coupler which is made of homogeneous medium

and the coordinate transformation employed by Xu is one special case of our transformation. The compact waveguide coupler can be realized with homogeneous uniaxial medium as long as the trapezoid region (connection part) is divided into several triangle blocks. Full wave simulation on the basis of finite element method is employed to verify the performance of the designed device. Simulation results indicate that the EM wave perfectly couples from one rectangular waveguide to another without causing any reflection. The coupler holds great promise in the application of integrated photonic circuit.

## II. TRANSFORMATION PRINCIPLE OF WAVEGUIDE COUPLER

The geometry structure of the waveguide coupler is presented in Fig. 1. Different from the coordinate transformation employed in Refs [12-14], we just need to map four triangular blocks AOB, BOM, MON, and AON in the virtual space ( $x, y, z$ ) into four corresponding areas AEB, BEC, CED, and AED in the physical space ( $x', y', z'$ ). As a result, only homogeneous and anisotropic medium are required to realize the coupler experimentally. If we define four blocks AEB, BEC, CED and AED as Region I, II, III and IV, the corresponding transformation functions can be expressed as follows,

$$\begin{aligned} \text{(I)} \begin{cases} x' = a_{11}x + b_{11}y + c_{11} \\ y' = a_{12}x + b_{12}y + c_{12} \\ z' = z \end{cases} & \text{(II)} \begin{cases} x' = a_{21}x + b_{21}y + c_{21} \\ y' = a_{22}x + b_{22}y + c_{22} \\ z' = z \end{cases} \\ \text{(III)} \begin{cases} x' = a_{31}x + b_{31}y + c_{31} \\ y' = a_{32}x + b_{32}y + c_{32} \\ z' = z \end{cases} & \text{(IV)} \begin{cases} x' = a_{41}x + b_{41}y + c_{41} \\ y' = a_{42}x + b_{42}y + c_{42} \\ z' = z \end{cases} \end{aligned} \quad (1)$$

$a_{i1}, b_{i1}, c_{i1}, a_{i2}, b_{i2},$  and  $c_{i2}$ , can be easily derived. It can be found that they only depend on the coordinate value of A, B, C, D, E, M, N, and O, respectively. Here,  $i=1, 2, 3$  and  $4$  represent Region I, II, III, and IV, respectively.

Then, based on the transformation principle, the material parameters in the four regions can be calculated

$$\overline{\overline{\epsilon}}' = \overline{\overline{\mu}}' = \Lambda \bullet \Lambda^T / \det(\Lambda) \quad (2)$$

in which  $\Lambda_i = (a_{i1}, b_{i1}, 0; a_{i2}, b_{i2}, 0; 0, 0, 1)$ , and  $\det(\Lambda_i) = a_{i1}b_{i2} - a_{i2}b_{i1}$ ,  $i=1, 2, 3, 4$ , which represents Region I, II, III and IV.

It is obvious that all constitute tensors are constants. Therefore, the material parameters are homogeneous and anisotropic.

## III. SIMULATION RESULTS AND DISCUSSIONS

In this part, full wave simulation is carried out to indicate the performance of the device. The boundary conditions on the left and right side of waveguide are set as perfect matched layer. The left and right boundary conditions are defined as scattering

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boundary condition. The other outer boundary conditions are set as perfect magnetic conductor. The incident wave propagates from left to right along x axis. The operating frequency of incident wave is 3 GHz. Without loss of generality, we only focus on the transverse electric (TE) incident wave with electric field polarized along z axis. The coordinate values of A, B, C, D, M, and N, are (0, 0.5), (0, -0.5), (0.5, -0.2), (0.5, 0.2), (0.5, -0.5), and (0.5, 0.5), respectively.

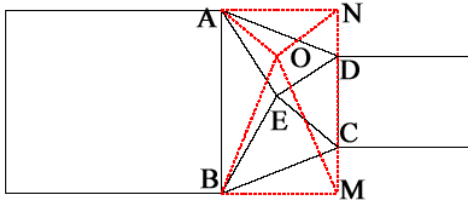


Figure 1 (Color online) Schematic structure of waveguide coupler.

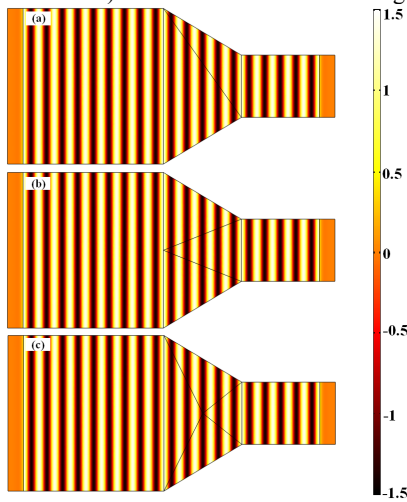


Figure 2 (Color online) Spatial distribution of electric field for different waveguide coupler when the trapezoid region is divided into two, three or four triangle blocks.

Electric field distribution of the device with different sets of material parameters are illustrated in Fig. 2. The trapezoid area consists of two, three, and four triangular regions, corresponding to the cases of Fig. 2(a) to (c). If the point E coincides with the point C, and simultaneously points O and M are overlapped, the coupler composes of two blocks. From Fig. 2(a), it can be seen that the EM wave perfectly couples from left waveguide to right waveguide without any reflection and energy loss. When the points O and E are overlapped and locate at the middle point of AB, as shown in Fig. 2(b), the device also performs well in transmitting EM field from left to right without causing any reflection. Here, it is worthy of pointing out that the device is degenerated into the case presented in Ref [15] when O (or E) locates at the side CD. Fig. 2(c) presents the electric field distribution of coupler when the points O and E are still overlapped but located in region ABCD. The device also has superior performance. Besides, the EM field inside the trapezoid area can be compressed and stretched without changing the functionality of the device as long as the coordinate values of M and N are varied.

#### IV. CONCLUSION

In summary, we introduce a general transformation to design the compact waveguide coupler. With the transformation, the coupler can be realized with homogeneous and anisotropic medium, which greatly reduces the difficulty of practical implementation of the device. Full wave simulation base on finite element method is performed to consolidate the good performance of the coupler.

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