

# Control, Routing and Mode-Locking Generation of Light Bullets in Planar Waveguide Arrays

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**Abstract**—A theoretical model of the mode-locking of light bullets in a planar slab waveguide array geometry is presented. The model yields three-dimensional localized light bullets that act as global attractors for particular parameter values. These light bullets can be controlled via non-uniformities in the gain applied to the array. This manipulation is robust and allows for bullet routing as well as the production of the NAND and NOR logic gates.

**Keywords**—component; mode-locking, waveguide arrays, light-bullets

## I. INTRODUCTION

The technological feasibility and nonlinear properties of semiconductor waveguide arrays (WGAs) make them an ideal technology for all-optical signal processing applications. The property of WGAs that make them so attractive from a technological standpoint is that nonlinear self-focusing is capable of overcoming discrete spatial diffraction for a sufficiently intense electrical field. This was predicted theoretically by Christodoulides and Joseph [1] and later shown experimentally by Eisenberg et al [2]. Based on this work, the WGA was proposed as an ideal component for both optical routing and switching purposes [2], temporal mode-locking of lasers [3], and the generation of spatial optical solitons [1,2].

In this manuscript, the generation of three-dimensional spatial confinement is based on the generation of spatial solitons in the WGA structure. Due to the planar structure of the waveguides in this slab waveguide array mode-locking model (SWGAML), the nonlinear mode coupling that creates temporal solitons in the WGA will generate the spatial confinement needed for light-bullet formation. In addition to the generation of bullets, we propose further enhancements to the SWGAML that allow the control and routing of bullets produced in the slab waveguide structure.

## II. GOVERNING EQUATIONS

Mode-locking in waveguide arrays is created by a competition between the saturable absorption generated by the nonlinear mode coupling [4] of the waveguides and the bandwidth limited gain. The waveguide array mode-locking model (WGAML) describes the temporal mode-locking in traditional ridge waveguide arrays. To model this slab

waveguide system, the WGAML was heuristically extended from one to two spatial dimensions [5]:

$$i\frac{\partial A_0}{\partial t} + \frac{D}{2}\nabla^2 A_0 + \beta|A_0|^2 A_0 + CA_1 + i\gamma_0 A_0 - ig(x, y, t)(1 + \tau\nabla^2)A_0 + i\rho|A_0|^4 A_0 = 0 \quad (1a)$$

$$i\frac{\partial A_1}{\partial t} + C(A_0 + A_2) + i\gamma_1 A_1 = 0 \quad (1b)$$

$$i\frac{\partial A_2}{\partial t} + CA_1 + i\gamma_2 A_2 = 0 \quad (1c)$$

where  $\nabla^2 = \partial_x^2 + \partial_y^2$ . The impact of current injection is modeled as a saturating gain:

$$g(x, t, y) = \frac{2g_0 f(x, y)}{1 + ||A_0||^2/e_0}. \quad (2)$$

In (1),  $A_0, A_1, A_2$  are the envelopes of the electric fields in the 0th, 1st, and 2nd waveguides respectively. Unlike the WGAML [4], the SWGAML is in a stationary frame and so  $D$  is the diffraction coefficient where the sign of  $D$  is the sign of the index of refraction.  $\beta$  determines the strength of the Kerr nonlinearity,  $\rho$  is proportional to the probability of three photon absorption occurring, the  $\nu_j$  are the aggregation of linear losses for each waveguide, and  $C$  is the strength of evanescent coupling between adjacent waveguides. The saturable gain  $g(x, y, t)$  accounts for the depletion of minority charge carriers at high optical intensities, resulting in a saturating gain. The filtering term,  $g\tau\nabla^2$  results in higher frequency spatial modes receiving lower amounts of gain than lower frequency modes. This term can be thought to arise from diffusion that results in more explicit models of the gain medium. This model supports light bullet formation from initial white-noise electric field excitations [5].

The function  $f(x, y, t)$  in (2) accounts for the possibility of non-uniform gain profiles. In order to uniquely specify the gain, it is imposed that the mean of  $f(x, y, t)$  is one at all times and  $f(x, y, t) > 0$ . Therefore, larger values of  $g_0$  always correspond to larger total injection currents regardless of the exact form of  $f(x, y, t)$ . The addition of non-uniform gain allows for a variety of additional dynamics not found in the uniform gain case. In particular, a non-uniform gain breaks

translational invariance in the system and creates solutions where the bullets translate in space.

### III. LIGHT BULLET ROUTING

The use of sloped gain generates a simple method of bullet routing. Specifically, light bullets gain velocity in the direction of the gradient of the gain. The simplest types of functions would be comprised of piecewise linear functions. These functions are capable of robustly routing bullets even through large angles. As an example, a junction can be created generating a single gain ramp but superimpose it with forbidden regions that contain no gain. Mathematically, a plus-shaped junction of the form

$$f(x, y, t) = \begin{cases} (1 + mx + ny) & |x| < 8 \text{ or } |y| < 8 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

where  $m$  and  $n$  control the direction in which the bullet moves and where one arm is the input and the other three outputs. Directing the light bullets is done simply by changing the direction of the slope of the gain along the plus-shaped junction. Figure 1 shows the three possible routings with this simple geometric application of the gain. The bullet location is chosen by application of gain gradient. When  $n=0.01$  the bullet is routed up, when  $n=0$  the bullet is routed across, and when  $n=-0.01$  the bullet is routed downward ( $m=0.01$ ).

In the application of a piecewise linear gain, the regions of zero gain in Figure 1 prevent the bullet from entering the region and do not destroy or trap the incoming bullet. The result is the bullet being routed through the junction rather than taking the most direct path between the outputs. The inclusion of these regions creates great flexibility in the generation and construction of devices used for bullet routing.

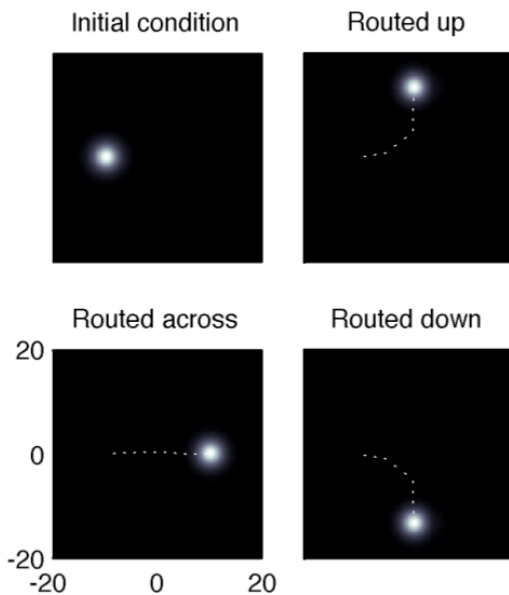


Fig. 1 Bullet routing using the gain equation in (3). The dotted lines show the location of the bullet center as time progresses. The piecewise linear gain routes the bullet through the junction.

### IV. GAIN MEDIATED INTERACTIONS: NOR AND NAND

It is also shown that the SWGAML is capable of supporting multiple-bullet solutions. By employing non-uniform gain, it is possible to route both of the bullets simultaneously. Therefore, it is possible to make use of multiple bullets and their interactions. The interaction of multiple bullets occurs through two distinct processes. The first is a direct interaction when two bullets are physically close enough to interact, similar to the interactions seen in the nonlinear Schrodinger equation. In the applications envisioned for this device, robust interactions can be gain-mediated. These types of interactions occur only through the gain term in (2). In the gain term, the level of saturation is determined by the energy. This non-local term allows bullets that are physically separated to influence one-another by increasing or reducing the gain of the system. While less powerful than direct interactions, this mechanism is still capable of producing both optical NOR and NAND gates.

### V. CONCLUSIONS

Light bullets hold tremendous potential as a critically important technology in the field of photonics. There are numerous technological methods both proposed and realized for engineering and controlling light bullets, and our approach is certainly not the only viable option for producing light bullets. However, as with all technologies, the implementation of light bullet technology requires the system to be both robust and inexpensive. Using slab waveguides, we have theoretically shown the ability of the SWGAML to produce and stabilize light bullets starting from noise. Furthermore, with the introduction of non-uniform gains these light bullets can be routed. Light bullets that are routable may be brought in close enough proximity to interact via gain. Gain mediated interactions are capable of reproducing the master logic gates and therefore all logic gates. Furthermore, the SWGAML architecture relies on simple input and output coupling as well as easily addressable routing via modulations of the gain provided to the system. Therefore, the SWGAML is able to control all-optical data streams and is capable of doing so in a feasible and easy to implement manner.

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