Modeling of pulse compressors using Kostenbauder matrices

Vikrant Chauhan, Jacob Cohen and Rick Trebino
837 State Street NW, Georgia Institute of Technology, Atlanta, GA-30332

Abstract- In this paper we present a method for modeling ultrashort-laser-pulse compressors/stretchers using Kostenbauder matrices. In this method, a Gaussian pulse is represented by a 2×2 complex Q-matrix and an optical element is represented by a 4×4 real K-matrix. This formalism models pulse compressors and performs full spatio-temporal analysis. Additionally, this formalism allows for uncertainty and sensitivity analyses of the compressors/stretchers. While being simple to implement numerically, this method is computationally much faster than the other equivalent approaches, such as use of Wigner matrices and Wigner functions.

I. INTRODUCTION

Pulse compressors are essential in all ultrashort-pulse laser labs. All regenerative amplifiers that use chirped-pulse amplification are reliant on pulse stretchers/compressors before/after the amplification process. Additionally, there are many applications that require the shortest duration of the pulse, such as multi-photon imaging, micro-machining and optical parametric amplification. This has motivated the use of a new class of extra-cavity pulse compressors, which can add tunable amounts of group-delay dispersion and thus provide control on the pulse width in an experiment[1, 2].

The stretching and compression of a pulse in a pulse compressor is brought about by introducing angular dispersion by using a dispersive device such as a grating, prism, or grism. Along with the angular dispersion, several other spatio-temporal distortions are introduced, which subsequently cancel inside the pulse compressor. Ideally, the net effect is only a stretched/compressed pulse in the output. Therefore modeling of pulse compressors/stretchers is an important task to determine the degree to which the pulse becomes distorted by the stretcher/compressor.

Most methods model only the temporal properties of the pulse. Neglecting the full spatio-temporal properties of a pulse can lead to an incorrect compressor design and an incomplete analysis of a pulse compressor. Therefore, these methods have serious limitations.

In our simulations, it is possible, and even easy, to model a pulse stretcher/compressor accurately and study the spatio-temporal distortions introduced by it due to minor misalignments. We also studied the evolution of spatio-temporal distortions on propagation, and they were found to worsen on propagation; distorting the pulse even more and thus rendering it useless for almost any application.

$$\begin{bmatrix} A & B & 0 & E \\ C & D & 0 & F \\ G & H & 1 & I \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow[]{\text{beam inversion}} \begin{bmatrix} A & B & 0 & -E \\ C & D & 0 & -F \\ -G & -H & 1 & I \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

II. K-MATRIX AND BEAM INVERSION

A general Kostenbauder matrix can represent all the essential optical elements, such as lenses, gratings, prisms, mirrors, medium interfaces, and composite elements, such as a grism, up to the first order. A matrix representing each of these devices can be constructed from the device parameters such as grating line density, apex angle of the prism, and refractive index of the material used, etc. The matrix for a series of optical elements is given by the matrix product of the individual K-matrices in the same order in which the elements are encountered by the pulse. In a pulse compressor, the angular dispersion introduced in the first pass through the dispersive element is removed by placing an identical and inverted dispersive element after it or by inverting the beam before the second pass. The latter is the case when a corner cube or a roof mirror is used to achieve inversion.

A general Kostenbauder matrix[3] and the effect of beam inversion on it, are shown in Eq.1. Each element of the general K-matrix represents a different spatio-temporal effect. A and D are magnifications, F is the angular dispersion, C and G are effective focal length and pulse-front tilt, respectively. B denotes position vs. slope. I represents GDD. E represents spatial chirp. And H represents time vs. angle.

III. PULSE PROPAGATION THROUGH A COMPRESSOR

In this section, we describe the modeling of a single grating pulse compressor (described in reference 2) and the propagation of a Gaussian pulse through it. Using the matrices for the grating on each pass through the compressor after accounting for the beam inversion, the K-matrix for a grating compressor can be derived as shown in Eq.2.

$$K = K_{grating} K_{space} K_{grating} K_{mirror} K_{grating} K_{space} K_{grating} \quad (2)$$

A Gaussian pulse is used as the input and is described in terms of the elements of the complex Q-matrix (Eq. 3) as follows (Note: $Q_{21}^{-1} = -Q_{12}^{-1}$)

$$E(x,t) = \exp\left\{-i\frac{\pi}{\lambda}[Q_{11}^{-1}x^2 + 2Q_{12}^{-1}xt - Q_{22}^{-1}t^2]\right\} \qquad (3)$$

The propagated Q-matrix represents the output pulse and is calculated using the elements of the K-matrix [3] for the compressor as shown below (Eq. 4).

$$Q_{out} = \frac{\begin{bmatrix} A & 0 \\ G & 1 \end{bmatrix} Q_{in} + \begin{bmatrix} B & E/\lambda \\ H & I/\lambda \end{bmatrix}}{\begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix} Q_{in} + \begin{bmatrix} D & F/\lambda \\ 0 & 1 \end{bmatrix}} \qquad (4)$$

Using the output Q-matrix, virtually all the properties of the output pulse and the pulse compressor/stretcher can be studied. The Q-matrix in x-t domain can be simply transformed to x-ω, k-ω and k-t domains using analytical relations derived in [4], where 'k' is the transverse wave vector and 'ω' is the angular frequency.

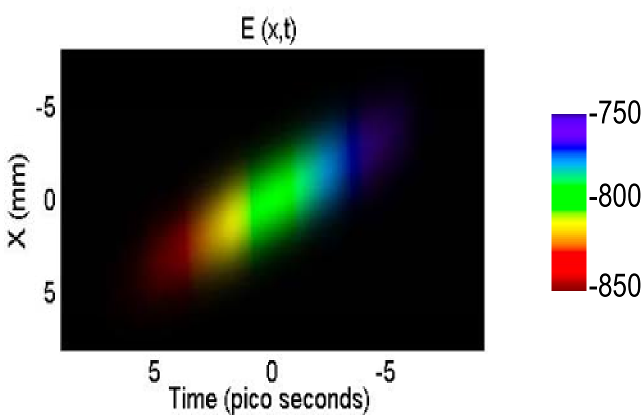

Fig. 1. Output of a misaligned compressor after propagation. Colors correspond to the constituent wavelengths of the pulse.

Thus all of the spatio-temporal intensity (and phase) distortions introduced by any first-order device can be calculated using this method. The results obtained from these simulations for the spatio-temporal distortions and the group-delay dispersion introduced by a single grating pulse compressor, were found in agreement with the experimental measurements and other simulations. The effects of angular misalignment and grating-separation mismatch in the conventional grating compressor (which uses two or four gratings) were also studied using this method. A transform-limited pulse of 100 fs width, centered at 800nm, was propagated through a conventional four-grating pulse compressor. In the simulation, the gratings were angularly misaligned by 2 degrees and the grating separation between the first and the second grating pair was offset by 1cm. This causes a large spatial chirp, pulse-front tilt, and angular dispersion in the output pulse. And these distortions become more severe after propagation by 10 meters as shown in Fig. 1.

IV. CONCLUSION

On propagation of the pulse, only the parameters of the Gaussian beam are changed, so it is possible to get very high resolution in the output without having over-sized arrays for space and time variables. Use of Wigner functions and Wigner matrices[5] can also, potentially, model the spatio-temporal characteristics of pulse compressors. The K-matrices can be transformed to Wigner matrices through a simple linear transformation. Therefore, the two methods are completely equivalent. However, using Wigner functions involves storing four dimensional Wigner functions, computing Fourier transforms and evaluating various marginals. This has an extremely high memory requirement and makes this method extremely difficult to use. The uncertainty analysis in our simulations is easy to implement. The input can obey some probability distribution, which leads to a characteristic variation in the output. For example, it can be shown that vibrations in chirped pulse amplification cause the output pulse width to display an L-shaped statistics (as both positive and negative separation mismatch lead to a longer pulse). Using these simulations we have also modeled prism- and grism-based pulse compressors and stretchers.

V. REFERENCES

[1] S. Akturk, X. Gu, M. Kimmel, and R. Trebino, "Extremely simple single-prism ultrashort- pulse compressor," Opt. Express **14**, 10101-10108 (2006).

[2] V. Chauhan, P. Bowlan, J. Cohen, and R. Trebino, "Single-diffraction-grating and grism pulse compressors," J. Opt. Soc. Am. B **27**, 619-624.

[3] A. G. Kostenbauder, "Ray-Pulse Matrices: A Rational Treatment for Dispersive Optical Systems," IEEE J. Quant. Electron. **26**, 1148-1157 (1990).

[4] S. Akturk, X. Gu, P. Gabolde, and R. Trebino, "The general theory of first-order spatio-temporal distortions of Gaussian pulses and beams," Optics Express **13**, 8642-8661 (2005).

[5] J. Paye and A. Migus, "Space time Wigner functions and their application to the analysis of a pulse shaper," J. Opt. Soc. Am. B **12**, 1480-1490 (1995).