Performance Evaluation of GRIN Lenses by Ray Tracing Method

Shuhei Yoshida, Shuma Horiuchi, Zenta Ushiyama, and Manabu Yamamoto

Tokyo University of Science
2641 Yamazaki, Noda City, Chiba 278-8510, Japan
e-mail: j8109703@ed.noda.tus.ac.jp

***Abstract*—To manufacture a gradient index lens (GRIN lens), the radial distribution of refractive index must be formed in the direction of the radius inside the lens. However, with some manufacturing methods, the distribution is not perfectly radial but slightly distorted. In order to evaluate the performance of GRIN lenses, we used the ray tracing method to measure the modular transfer function (MTF).**

I. Introduction

Cylindrical GRIN lenses, which are used as the optical information reading element in scanners, copy machines and so forth, are also called “rod lenses”, and their refractive index distribution decreases in a quadric manner along the radius from the central axis to the circumference. There are several methods for manufacturing GRIN lenses, but some produce a somewhat distorted distribution of refractive index instead of the ideal distribution. It is therefore necessary to evaluate the image-forming performance of GRIN lenses. In this study, we used the ray tracing method to obtain the point spread function, and measured the MTF that quantitatively expresses the reproducibility of optical systems, in order to evaluate the image-forming performance of GRIN lenses.

II. New methods of manufacturing plastic GRIN lenses

There are several methods for manufacturing GRIN lenses, and in this study we used the continuous lamination and shape-forming method, the principle of which is shown in Figure 1.

A plastic fiber with specified refractive index is pulled out in the direction opposite to gravity from the free surface of a polymer solution with a refractive index slightly lower than that of the fiber. The polymer solution adheres to the fiber surface as a thin film. During a certain retention time in this condition, the monomer element inside the polymer solution adhering to the fiber diffuses into the solid fiber, forming a refractive index distribution near the interface between fiber and solution. When the monomer concentration reaches the specified concentration distribution, ultraviolet rays are irradiated to polymerize the monomer and fix the distribution of refractive index. By pulling this fiber from the free surface of a polymer solution with a lower refractive index, a thin solution film is formed in a similar fashion. The monomer in solution diffuses into the polymerized inner thin film and forms a new distribution of refractive index. This procedure is repeated several times to obtain a GRIN fiber with a parabolic distribution of refractive index from the central axis to the circumference and of the desired diameter [1][2][3].

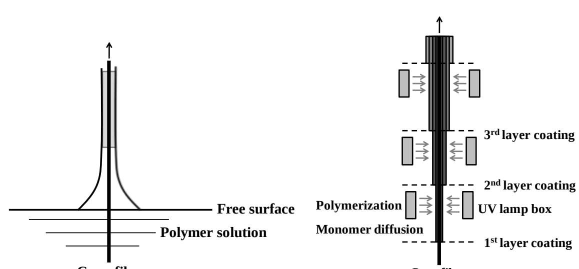

Fig. 1. GRIN lens manufacture by continuous lamination and shape-forming

III. Simulation method for evaluating GRIN lenses

A. Ray tracing method

We consider a light beam from the point P_1 to the point P_2. According to Fermat's principle, it is expressed as follows:

$$\delta\int_{P_1}^{P_2} n(r)ds = 0. \quad (1)$$

where *s* indicates the unit vector of the direction of travel of the light beam, *n* indicates the refractive index, and *r* indicates the position vector on the light beam.

By expressing the position vector on the light beam “*r*” with the orthogonal coordinate system, and resolving it to the Euler–Lagrange equation for formulation, we obtain four simultaneous differential equations:

$$\dot{y} = \frac{p_y}{\left(n^2 - p_y^2 - p_z^2\right)^{1/2}}. \quad (2)$$

$$\dot{z} = \frac{p_z}{\left(n^2 - p_y^2 - p_z^2\right)^{1/2}}. \quad (3)$$

$$\dot{p}_y = -\frac{n(\partial n / \partial y)}{\left(n^2 - p_y^2 - p_z^2\right)^{1/2}}. \quad (4)$$

$$\dot{p}_z = -\frac{n(\partial n / \partial z)}{\left(n^2 - p_y^2 - p_z^2\right)^{1/2}}. \quad (5)$$

where p_y is the generalized momentum in the *y* direction, and p_z is the generalized momentum in the *z* direction. By solving these simultaneous differential equations with the Runge-Kutta method, the trajectory of the light beam within the distributed refractive index medium can be analyzed.

978-1-4244-7017-4/10/$26.00 ©2010 IEEE

B. MTF *measuring method*

To obtain a quantitative value for the reproducibility of optical systems, the Fourier spectrum in a given frequency is divided by the Fourier spectrum at zero frequency and the result is normalized. This is defined as the "spatial frequency transfer function" or "OTF (Optical Transfer Function)". Figure 2 shows the principle.

An eye point is divided into N rectangles of equal area, and the coordinates of the arriving image surface of the light beam that passes through the top of the jth grid is assumed to be (x_j, y_j), and $A = Ndudv$:

$$\mathrm{OTF}(s,t) = \frac{1}{Ndudv}\sum_{j=1}^{N} \exp\{-2\pi i(sx_j + ty_j)\} dudv = \frac{1}{N}\sum_{j=1}^{N} \exp\{-2\pi i(sx_j + ty_j)\}. \quad (6)$$

where s is the spatial frequency in the sagittal direction, and t is the spatial frequency in the meridional direction.

The extracted absolute value of OTF is called "MTF", and measuring MTF allows the image-forming performance between two points to be evaluated.

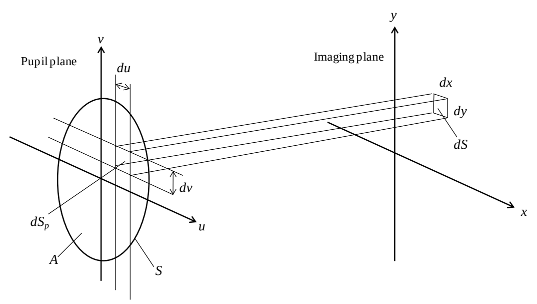

Fig. 2. Outline of geometrical-optical intensity distribution

IV. EVALUATION RESULTS

We took a manufactured GRIN lens as a sample and measured its MTF. Figure 3 shows the measurement result obtained by examining the distribution of refractive index in the GRIN lens with an interference microscope. Figure 4 shows a spot diagram based on the analysis of the path of the light beam entering a GRIN lens having the distribution of refractive index shown in Figure 3. Table 1 shows the result of MTF measurement based on the spot diagram shown in Figure 4.

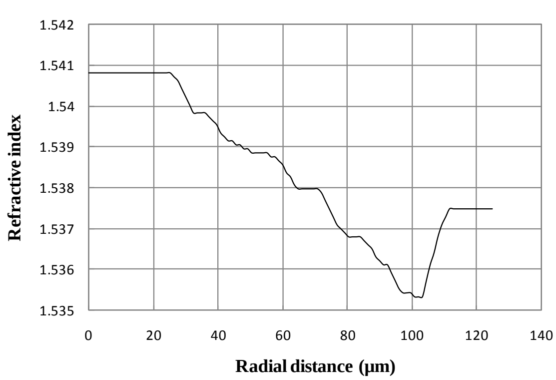

Fig. 3. Distribution of refractive index in GRIN lens

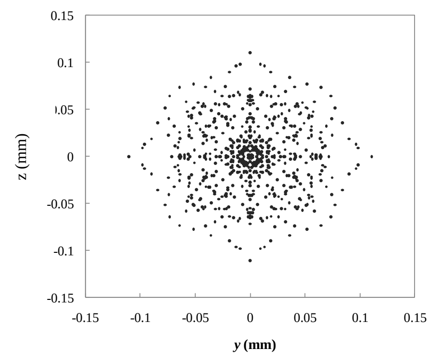

Fig. 4. Spot diagram

TABLE I
MTF MEASUREMENT RESULT

	Spatial frequency	6LP/mm
MTF (%)	Test result	45
	Simulation result	44

V. CONCLUSION

In this study, we demonstrated that the point spread function could be obtained by analyzing a light beam entering a GRIN lens with the ray tracing method. Measurement of MTF based on the point spread function enabled the image-forming performance of the GRIN lens to be evaluated.

REFERENCES

[1] M. Oda, N. Kubota, N. Kitahashi, N. Sugawara, S. Suga, H. Yoshii, T. Furuta, "Innovative Multi-Layer Coating Method for Preparation of Gradient Refractive Index Lens," Proc. 11th APPChE Congress, Paper ID 119, Kuala Lumpur, Malaysia (2006)

[2] M. Oda, S. Suga, H. Yoshii, T. Furuta, "Multilayer coating by drawing a thin plastic fiber through a polymer solution," *Asia-Pacific J. Chem. Eng.*, 3, pp.63-69 (2008)

[3] M. Oda, S. Suga, H. Yoshii, T. Furuta, "Multi-layer coating by continuous withdrawal of a thin plastic fiber through polymer solution," *Kagaku Kogaku Rombunshu*, 34, pp.187-193 (2008)