

# Mesh-free Spectral Method Analysis of Optical Waveguides and Wave Propagation

Fahhad Alharbi

King Abdulaziz City for Science and Technology  
 IBM Almaden Research Center  
 Email: alharbif@kacst.edu.sa

J. Campbell Scott

IBM Almaden Research Center

**Abstract**— A new, fast, and efficient full vectorial multi-domain spectral method (MDSM) to simulate optical devices is presented. The presented method is mesh-free and is used for modal analysis of optical waveguides and to simulate optical wave propagation and scattering in two dimensional structures. As common to spectral methods, the presented method is very fast and accurate. Simulations of some complex structures were performed in less than a second using personal computer.

## I. INTRODUCTION

The optics industry has reached sophisticated levels of fabrications, applications, and miniaturization. This has increased the demand for fast and consistent simulation tools. Such tools are required to be applicable to arbitrary structures of complex geometries and wide-ranging material properties. Also, these tools should be capable of large scale applications.

With the growing complexities of numerically studied problems, the spectral method starts gaining more attention mainly because of its high level of analyticity. This reduces the computational memory and time requirements where a major part of the problem is solved analytically. Spectral methods are a special family of weighted residual methods where the unknown functions are approximated by either a functional expansion of or interpolation (collocation method) using a preselected basis set. Functional expansion is used in this work. In MDSM, the computational window is divided into domains where the structural parameters are smooth and the discontinuities lie at the boundaries [1]–[3]. This domain division is used to avoid the Gibbs phenomenon, which is associated with discontinuities of structural functions. Then, the spectral method is applied in each domain to build the numerical matrices and vectors. These are then joined by applying the proper boundary conditions between domains. Structural functions are problem dependent. In this work, the structural function is the dielectric constant.

The presented method is applicable for 3D structures that are invariant in one direction (assumed to be the  $z$ -direction). For the variant cross-section of the structure and as standardly done in dimension reduction methods, the structure function is divided into  $M$  regions in such a way that in each region, the cross-section is invariant in another dimension ( $x$  axis). Then the regions are divided into  $N$  layers. So, as shown in Fig.1, the computational window is divided into domains where the dielectric properties, which are the structural functions, are homogeneous in each domain. Cartesian coordinates are assumed and used in this work although any other 2D system can be used.

Since the domains are defined such that each one is homogeneous, the governing harmonic wave equation in each domain is simply Helmholtz equation, which is

$$\frac{\partial^2 E_{mn}}{\partial x^2} + \frac{\partial^2 E_{mn}}{\partial y^2} + k_0^2 (\varepsilon_{mn} - n_z^2) E_{mn} = 0 \quad (1)$$

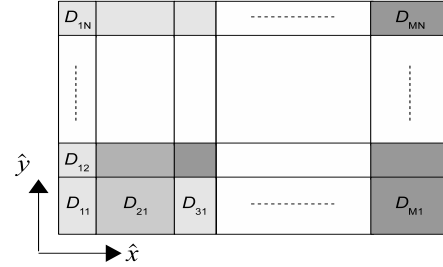


Fig. 1. The division of the structure into domains

for the  $mn^{th}$  domain where  $k_0$  is the free space wavenumber and  $\varepsilon_{mn}$  is the dielectric constant in this domain.  $n_z$  is effective propagation index in the  $z$  direction. In the case of wave propagation analysis, it represents the projection in  $z$  direction of the input field wavenumber. In modal analysis it is the effective refractive index of the mode.

In the presented multi-domain approach, the wave function and mode profile are approximated by 1D known basis set in one axis and the approximating coefficients are set as functions of the second dimension as follows

$$E_{mn} = \sum_l c_{mnl}(x) P_{mnl}(y) \quad (2)$$

where  $P_{mnl}(y)$  are the preselected basis functions. By applying boundary conditions of the base field components in all the layers in the same region and by combining their corresponding matrices, we obtain the following differential system that represents the wave equation in each region

$$\mathbf{A}_m \frac{d^2 \mathbf{c}_m}{dx^2} + \mathbf{B}_m \frac{d \mathbf{c}_m}{dx} + \mathbf{C}_m \mathbf{c}_m = 0 \quad (3)$$

This equation can be solved in many ways. In this work, it is solved using reduced eigenvalues and eigenvectors sets. So, the solution of this equation is simply

$$c_m(x) = \mathbf{R}_m^f e^{\Lambda_m^f(x-x_m)} \mathbf{f}_m + \mathbf{R}_m^b e^{\Lambda_m^b(x-x_{m+1})} \mathbf{b}_m \quad (4)$$

where  $\Lambda_m^f$  ( $\Lambda_m^b$ ) and  $\mathbf{R}_m^f$  ( $\mathbf{R}_m^b$ ) are the forward (backward) traveling eigenvalues and their corresponding eigenvectors matrices. Eigenvalues are arranged as forward and backward traveling to ensure that the exponentials are not diverging. The only unknowns in this solution are  $\mathbf{f}_m$  and  $\mathbf{b}_m$ . These shall be found after applying vertical boundary and initial conditions.

## II. SOME RESULTS

### A. Multimode Interference (MMI) Switch wave propagation

The method has been used to simulate optical wave propagation in MMI switch, recently presented by Yin et al. [4]. The structure

of the switch is shown in Fig.2, where the switching of the field is managed by changing nonlinearly the index in the shaded region. If the index of the shaded area is set to  $n_2$ , the field is guided into *Output A* ('on' state), and it is guided into *Output B* ('off' state) if the shaded index is equal to  $n_1$ . The switch was analyzed ([4]) using 2-D beam propagation method (BPM) at  $\lambda = 1.55\mu m$ . Fig.3 and Fig.4 show TE light intensities obtained by the presented method and by BPM [4] for 'on' state and 'off' state respectively. The calculated optical crosstalk is 25.3 dB. As can be seen from the figures, the wave propagation simulation is similar for both states and by using the two different methods.

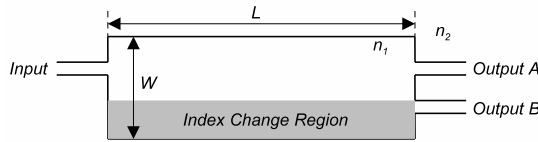


Fig. 2. The structure of the studied MMI [4]

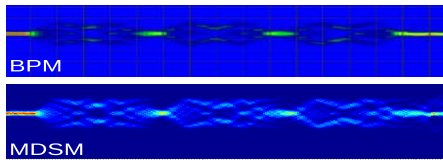


Fig. 3. Wave propagation in 'on' state; BPM [4] (top) and MDSM (bottom)

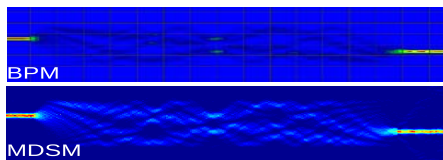


Fig. 4. Wave propagation in 'off' state; BPM [4] (top) and MDSM (bottom)

*B. Nano scale grating scattering*

Nano scale molecular optoelectronics have attracted interest in the recent years. For these devices, an accurate optical field distribution calculation is essential initial step. We used the presented method in our work in this area. It exhibit a superior performance both in time and accuracy when compared with RCWA software. Fig.5 compared vectorial distribution for a wave at normal incidence on a nano optical grating using the presented method with RCWA. In this analysis,  $E_x$  field is a result of diffraction of  $E_y$  component.. As can be seen, the two methods highly agree.

*C. Modal analysis of multimode wide fully etched waveguide*

Wide fully etched waveguide structures (Figure-6) are used as multimode waveguides. Such structure allows many modes, where the separations between the effective indices are very small and are difficult to detect. The presented method obtains them easily. The used parameters are  $w = 10 \mu m$ ,  $h = 0.5 \mu m$ ,  $n_s = 1.95$ ,  $n_c = 2.3$ , and  $n_{cl} = 1$  at  $1.30 \mu m$  wavelength. The contours of amplitudes of the 11 allowed modes are shown in Figure-7.

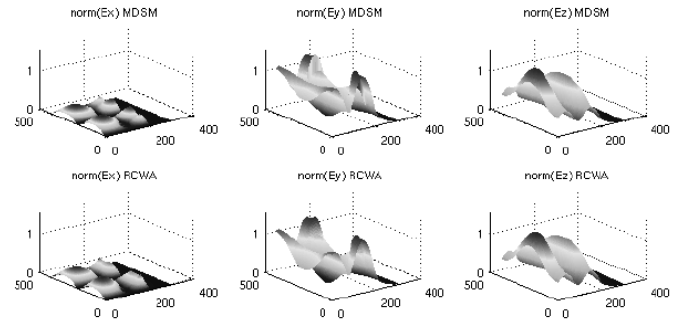


Fig. 5. A comparison between the presented method and RCWA

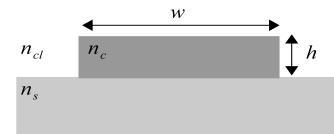


Fig. 6. Wide fully etched waveguide geometry

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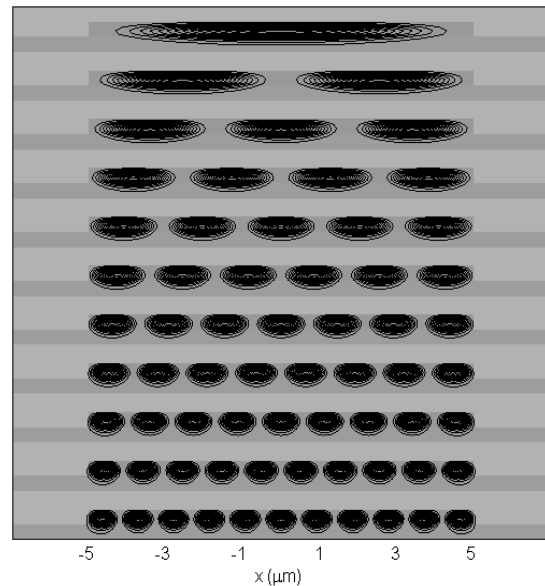


Fig. 7. The contours of the amplitudes of the allowed modes in the studied fully etched waveguide