

# Non-paraxial beam propagation in nonlinear optical waveguides using complex Jacobi iteration

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**Abstract**—We present the recently introduced beam propagation method using complex Jacobi iteration adapted for efficient modeling of non-paraxial beam propagation in nonlinear optical waveguides.

## I. INTRODUCTION

AN external excitation of a nonlinear waveguide was demonstrated to produce spatial multisoliton emission from the waveguide, in which a sequence of bright solitons could be emitted for sufficient input power [1]. For waveguides with cladding made by nonlinear material where the refractive index depends on field intensity, it is difficult to obtain analytical solutions and thus, computational efforts are necessary for design and modeling of these kinds of nonlinear optical waveguide devices. There are several methods developed to simulate the optical propagation in nonlinear waveguides in which the beam propagation method (BPM) has become one of the most powerful and efficient techniques.

A great number of BPM versions including the finite element-based and the finite difference-based BPM have been developed for these kinds of problem [2-3]. Those methods normally used the traditional direct matrix inversion (DMI) to find a solution of a propagation equation between two successive propagation steps. However, for large structures with long propagation paths and varying boundaries along propagation direction, the DMI technique is numerically very intensive.

To overcome this problem, recently we proposed the iterative BPM where at each propagation step the beam propagation is recast in terms of a Helmholtz equation with source term and then solved effectively by the recently introduced complex Jacobi iteration (CJI) [4].

Furthermore, by introducing an extra calculation step the CJI method has been successful employed to simulate photonic components composed by materials with nonlinear Kerr effects [5]. Therefore, by doing the same procedure for dealing with nonlinear effects at each propagation step, our

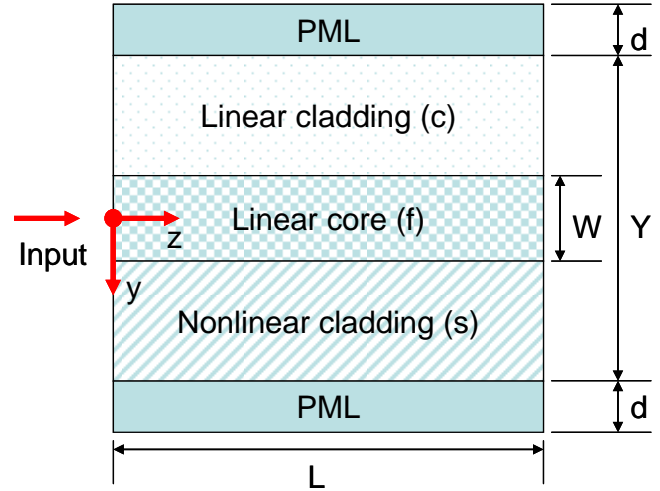


Fig. 1. Nonlinear optical waveguide with a linear core bounded by linear and nonlinear claddings.

recently proposed iterative BPM is extended and is very competitive for modeling optical propagation in nonlinear optical waveguides.

## II. FORMULATIONS

The scalar non-paraxial beam propagation equation based on the modified Pade(1,1) approximant operator is given by [4]

$$(1 + \xi P)\Psi^{n+1} = (1 + \xi^* P)\Psi^n \quad (1)$$

$$\text{where } P = \nabla_{\perp}^2 + k_0^2(n^2 - n_{ref}^2) = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_0^2(n^2 - n_{ref}^2),$$

$k = k_0 n_{ref}$ ,  $n_{ref}$  is the reference refractive index,

$$\xi = \frac{1}{4k^2(1 + i\beta/2)} - \frac{i\Delta z}{4k}, \quad \xi^* = \frac{1}{4k^2(1 + i\beta/2)} + \frac{i\Delta z}{4k}, \quad \beta$$

is a damping parameter,  $\Delta z$  is the propagation step size and  $n$  is the intensity dependent refractive index as follows [1]:

$$n^2 = n_l^2 + \gamma_l |\Psi|^2, \quad l = c, s, f. \quad (2)$$

with the subscripts c,s,f referring to the cladding, substrate and film, respectively.

By dividing both sides of Eq. (1) by  $\xi$ , we obtained

$$[\nabla_{\perp}^2 + k_0^2(n^2 - n_{ref}^2) + \frac{1}{\xi}]\phi^{n+1} = (\frac{\xi^*}{\xi}P + \frac{1}{\xi})\phi^n \quad (3)$$

or

$$[\nabla_{\perp}^2 + k_0^2(n^2 - n_{ref}^2) + \frac{1}{\xi}]\phi^{n+1} = source\ term. \quad (4)$$

It is clearly seen that the beam propagation equation can be recast as Helmholtz equation with source term in an effective medium with loss determined by the imaginary part of  $\frac{1}{\xi}$ .

Therefore, at each propagation step the beam propagation equation will be solved effectively by the CJI method with adding an extra calculation step for dealing with nonlinear effect [5].

### III. NUMERICAL RESULTS

We consider a nonlinear optical waveguide with a linear core bounded by linear and nonlinear claddings as shown in Fig. 1, where the refractive index  $n_s = n_c = 1.55, n_f = 1.57, \gamma_s = 0.01, \gamma_c = \gamma_f = 0, W = 5 \mu m, Y = 50 \mu m, d = 1 \mu m$  and the optical wavelength  $\lambda = 1.3 \mu m$ . The fundamental TE mode as excitation is launched into the waveguide. The beam propagation in linear optical waveguide is shown in Fig. 2 (a), while the evaluation of input beam in nonlinear optical waveguide is depicted in Fig. 2(b). From Fig. 2(b), the soliton is emitted through the film cladding interface into the nonlinear cladding and propagates away from it. The calculated results are in very good agreement with those obtained by the other authors [1,2]. For a small propagation step size, we found that the amount of effective absorption is very high. This is condition that favors rapid convergence of CJI. With a very strict propagation error tolerance of  $10^{-9}$ , the CJI method required about 24 iterations and 0.035 seconds to obtain a solution between two successive propagation steps, while the direct matrix inversion performed the propagation in 0.365 seconds, around ten times slower than the CJI method.

### IV. CONCLUSION

The recently introduced BPM adapted for modeling of non-paraxial beam propagation in nonlinear optical waveguides has been presented. By typically choice of propagation step size, the iterative method obtains rapid convergence and is very competitive for demanding problems in comparison with the traditional direct matrix inversion for BPM.

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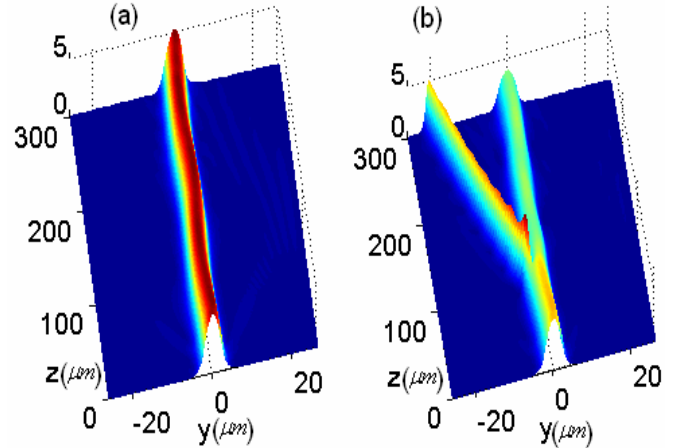


Fig. 2. (a) TE mode propagating in linear waveguide and (b) soliton emission in nonlinear waveguide.