Calculation of Light Delay by FDTD Technique and Padé Approximation

Yong-Zhen Huang and Yue-De Yang
State Key Laboratory on Integrated Optoelectronics, Institute of Semiconductors, Chinese Academy of Sciences, PO Box 912, Beijing 100083

***Abstract-* The time delay for light transmission in a coupled microring waveguide structure is calculated from the phase shift of the transmission coefficient obtained by Padé approximation with Baker's algorithm from FDTD Output. The results show that the Padé approximation is a powerful tool for saving time in FDTD simulation.**

I. Introduction

Recently, slow light has attracted great attentions due to its potential applications for optical data buffering in optical signal processing, optical interconnection, and optical computing, etc. Coupled-microring-resonator waveguides and photonic crystal waveguides were investigated to realize the slow light in photonic integrated circuits. Transfer matrix method was widely applied to analyze the light transmission and light delay for coupled microring waveguide structure. The finite-difference time-domain (FDTD) technique is a powerful tool for simulating light transmission in complex optical micro-structures. The FDTD simulation only yields the time variation of electromagnetic fields, which should be transformed into frequency-domain to obtain transmission spectrum. The natural selection is the fast Fourier transform (FFT) method, which has the resolution inversely proportional to the total persistence time of the FDTD iteration. To save the terrible burden of the time consumption FDTD process, we have introduced the Padé approximation with Baker's algorithm to transfer the FDTD output from time domain to frequency domain for optical microcavities [1,2]. The Padé approximation were also applied to calculate propagation loss for photonic crystal waveguide and high Q photonic crystal microcavities [3,4]. Only the amplitude of transmission spectrum is required in calculating the mode frequencies and Q-factor. In this article, we apply the Padé approximation to evaluate the light delay in microring coupled waveguide structures from the phase shift spectrum obtained by the Padé approximation from the FDTD output.

II. Padé Approximation With Baker's Algorithm

In this section, we present the basic formulae for the Padé approximation with Baker algorithm as in [1]. According to the definition of Padé approximants, a given power series $f(z)=\sum_{n=0}^{\infty}c_n z^n$ can be approximately expressed as

$$\sum_{n=0}^{\infty}c_n z^n - \frac{P(z)}{Q(z)} = O(z^{M+N+1}) , \quad (1)$$

where $P(z)=\sum_{n=0}^{M}a_n z^n$ and $Q(z)=1+\sum_{n=1}^{N}b_n z^n$. $P(z)/R(z)$ is defined as Padé approximant $[M, N]_{f(z)}$ of the given power series $f(z)$. Assuming $S(n\Delta t)$ is the time response of one of electromagnetic field components, where n = 0, 1, …, ∞ is the sampling number and Δt is the sampling interval in the FDTD simulation, we can obtain field spectrum from the FDTD time sequences by FFT method:

$$U(\infty, f) = \sum_{n=0}^{\infty} S(n\Delta t)\exp(-i2\pi f n\Delta t) . \quad (2)$$

We can define a power series

$$F(z,f) = \sum_{n=0}^{\infty} C_n z^n , \quad (3)$$

with $C_n = S(n\Delta t)\exp(-i2\pi fn\Delta t)$ and have $F(1,f) = U(\infty,f)$. In real FDTD simulation, we only get a finite time sequence of the field $S(n\Delta t)$ with n = 0, 1, …, N. Assuming N is an even number and applying the Padé approximant $[N/2, N/2]_{F(z,f)}$ to the given power series (3) at z = 1, we can obtain the approximation of $U(\infty, f)$. According to Baker's algorithm, the Padé approximant $[N/2,N/2]_{F(z,f)}$ can be calculated by

$$\frac{\eta_{2j}(z)}{\theta_{2j}(z)} = [N-j, j]_{F(z)} , \quad (4)$$

$$\frac{\eta_{2j+1}(z)}{\theta_{2j+1}(z)} = [N-j-1, j]_{F(z)} , \quad (5)$$

which satisfy the following recursion relations

$$\frac{\eta_{2j+2}(z)}{\theta_{2j+2}(z)} = \frac{\overline{\eta}_{2j+1}\eta_{2j}(z) - z\overline{\eta}_{2j}\eta_{2j+1}(z)}{\overline{\eta}_{2j+1}\theta_{2j}(z) - z\overline{\eta}_{2j}\theta_{2j+1}(z)} , \quad (6)$$

$$\frac{\eta_{2j+3}(z)}{\theta_{2j+3}(z)} = \frac{\overline{\eta}_{2j+2}\eta_{2j+1}(z) - \overline{\eta}_{2j+1}\eta_{2j+2}(z)}{\overline{\eta}_{2j+2}\theta_{2j+1}(z) - \overline{\eta}_{2j+1}\theta_{2j+2}(z)} , \quad (7)$$

where $\overline{\eta}_j$ is the coefficient of the highest power of z in $\eta_j(z)$. The initial values are $\eta_0 = \sum_{n=0}^{N} C_n z^n$, $\eta_1 = \sum_{n=0}^{N-1} C_n z^n$, θ_0=1, and θ_1=1. The field amplitude at a given frequency can be calculated through the recursion relation with the initial values based on finite time sequences of FDTD output. The ratio of transmission field amplitude to the input field amplitude is the transmission spectrum

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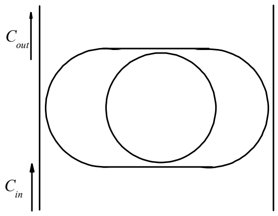

Fig. 1. The embedded ring resonator of a racetrack and inside ring coupled to two straight waveguides.

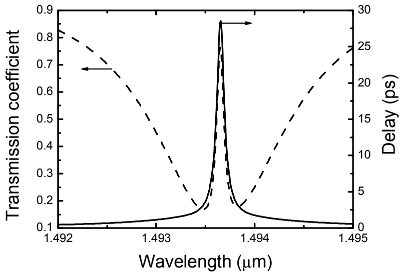

Fig. 2. Transmission and time delay spectra obtained by the FDTD simulation and the Padé approximation.

$$T(\omega) = A(\omega)\exp(-i\phi)\,, \tag{8}$$

where $A(\omega)$ and $\phi(\omega)$ are transmission amplitude and phase shift. The time delay can be calculated by

$$\tau(\omega) = \frac{d\phi}{d\omega}\,. \tag{9}$$

III. NUMERICAL RESULTS

We calculate the transmission spectrum for the embedded ring resonators as shown in Fig. 1, which was proposed in [5]. The radius and the length of the straight waveguide of the racetrack ring are 3 μm and 2.04 μm, respectively, the embedded microring radius is 2.6μm, and the waveguide width is 0.2μm with the refractive index of 3.2 surrounded by air. The space step is 10 nm, and the time step is chosen to 2.33×10^{-17}s satisfy the Courant condition. The transmission and time delay spectrum obtained from the FDTD output after light transmit time of 6 ps are plotted in Fig. 2. We have the same mode wavelength at mode number 37 in the racetrack and mode number 26 in the embedded microring by adjusting the parameters, which satisfies the difference of mode numbers to be an odd number [5]. The peak transmission is 0.8 and time delay is 28 ps at the resonance peak. Taking a continuous exciting source at the peak wavelength, we get the field pattern as shown in Fig. 3 after the light transmission of 20 ps, which mainly locates in the input waveguide side of the racetrack, the transmission is only 0.5 because the field distribution is not the steady pattern yet.

By FDTD simulation before the pulse transmission over one period inside the microring, we can calculate the coupling coefficients between the racetrack and input waveguide to be 0.15, and that between the two rings to be 0.11. Based on the obtained coupling coefficients, we calculate the transmission spectrum and time delay by transfer matrix method assuming

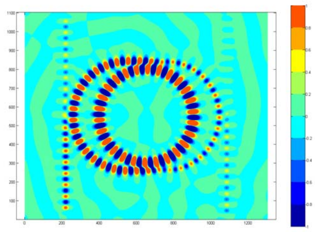

Fig. 3. Mode field pattern obtained after 20 ps FDTD simulation under injection of continuous wave at the resonance wavelength.

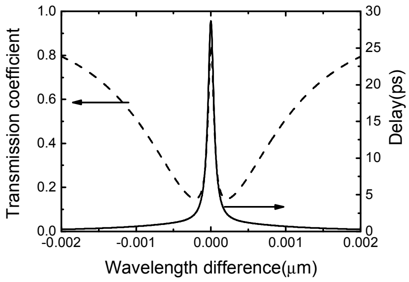

Fig. 4. Transmission and time delay obtained from the transfer matrix method with the coupling coefficients obtained by FDTD simulation.

the same resonance wavelengths for the two coupling rings and the transmission loss of 2 dB/cm. The obtain results are plotted in Fig. 4, which agrees very well with Fig. 2.

IV. CONCLUSIONS

The numerical results show that the Padé approximation is a powerful tool to evaluate the light delay from the FDTD output for complex optical waveguide structures for saving the FDTD computing time.

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